SOME

GENERAL THEOREMS

Of conſiderable uſe in the

HIGHER PARTS

OF

MATHEMATICS.

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By MATTHEW STEWART Miniſter at *Roſneath.*

EDINBURGH:
Printed by W. SANDS, A. MURRAY, and J. COCHRAN.
Sold by ſaid W. SANDS in the Parliament-cloſe, and by J. and
P. KNAPTON, at the Crown in Ludgate ſtreet, *London.*

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PREFACE.

THE theorems contained in the following ſheets are given without being demonſtrated, excepting the firſt five: And as they are entirely new, ſave one or two at moſt, the author expects their being publiſhed even in this way may be agreeable to thoſe that are not unaccuſtomed to ſpeculations of this kind. Such will eaſily allow, that to explain, in a proper way, ſo many theorems, ſo general, and of ſo great difficulty as moſt of theſe are, would require a greater expence of time and thought than can be expected ſoon from one in the author's ſituation. He therefore thought it was better they ſhould appear in the way they now are, than lie by him till an uncertain hereafter. If any give themſelves the trouble to explain ſome of theſe theorems, they will find their time and pains ſufficiently rewarded, by the diſcovery of ſeveral new and curious propoſitions that otherwiſe might have eſcaped their obſervation.

EDINBURGH, Oct. 1,
1746.

SOME

GENERAL THEOREMS

Of conſiderable uſe in

The higher parts of Mathematics.

PROPOSITION I. *Fig.* 1.

I*F from* A, *the vertex of any triangle* ABC, *there be drawn* AD *to any point* D *in the baſe, and* DE, DF *be drawn parallel to* AC, AB *meeting* AB, AC *in* E, F, *the ſum of the rectangles* BAE, CAF *will be equal to the ſquare of* AD *together with the rectangle* BDC.

About the triangle ABC let there be a circle deſcribed, and let AD meet the circle in G; join BG, CG; and from the point E draw EH making the angle AHE equal to the angle ABG, and produce AB to any point K.

Becauſe the angles AHE, ABG are equal, the points E, B, G, H are in a circle; therefore the rectangle BAE is equal to the rectangle GAH. The angle EHD will alſo be equal to the angle GBK, that is, to the angle ACG. And becauſe AC, DE are parallel, the angles EDH, GAC will be equal; therefore the triangles EDH, GAC will be ſimilar; and therefore AC will be to AG, as DH to DE: Therefore the rectangle contained by AC, DE, that is, the rectangle CAF, is equal to the rectangle contained by AG, DH. But becauſe the rectangle BAE is equal to the rectangle GAH, and likewiſe the rectangle CAF equal to the rectangle contained by AG, DH, the ſum of the rectangles BAE, CAF will be equal to the rectangle GAD, that is, equal to the rectangle ADG together with the ſquare of AD. But the rectangle ADG is [35. 3.] equal to the rectangle BDC; therefore the ſum of the rectangles BAE, CAF is equal to the ſquare of AD together with the rectangle BDC. *Q. E. D.*

PROPOSITION II. *Fig.* 2. 3.

In the right line AB *take any point* C, *between the points* A, B; *and from the points* A, B, C *let there be drawn right lines to any*

any point D; *the ſquare of* AD *together with the ſpace to which the ſquare of* BD *has the ſame ratio that* BC *has to* CA, *will be equal to the rectangle* BAC *together with the ſpace to which the ſquare of* CD *has the ſame ratio that* BC *has to* BA.

1. When the point D [*Fig.* 1.] is not in the line AB.

Draw AE, DF parallel to CD, AB meeting BD, AE in E, F.

Becauſe the ſquare of BD is to the rectangle BDE as BD to DE, that is, as BC to CA, the rectangle BDE will be the ſpace to which the ſquare of BD has the ſame ratio that BC has to CA. And becauſe the ſquare of AF, that is, the ſquare of CD, is to the rectangle EAF as AF to AE, that is, as BD to BE, or BC to BA, the rectangle EAF will be the ſpace to which the ſquare of CD has the ſame ratio that BC has to BA. But [1.] the ſquare of AD together with the rectangle BDE, is equal to the rectangle BAC together with the rectangle EAF; therefore the ſquare of AD together with the ſpace to which the ſquare of BD has the ſame ratio that BC has to CA, is equal to the rectangle BAC together with the ſpace to which the ſquare of CD has the ſame ratio that BC has to BA. *Q. E. D.*

2. When the point D [*Fig.* 2.] is in the line AB.

Draw CE perpendicular to AB, and let CE be equal to AC; join AE, BE; draw BF parallel to CE meeting AE in F; and draw DG parallel to CE or BF meeting AE, BE in G, H; and join GC, HC.

Becauſe AC is equal to CE, AD will be equal to DG; therefore the ſquare of AD will be equal to twice the triangle ADG. And becauſe the ſquare of BD is to the rectangle BDH, that is, twice the triangle BDH, as BC to CE, or CA, twice the triangle BDH will be the ſpace to which the ſquare of BD has the ſame ratio that BC has to CA. Again, becauſe AC, CE are equal, the rectangle BAC will be equal to twice the triangle AEB; and becauſe EG is to EF, that is, CD to CB, as GH to BF, or AB, CD will be to GH as BC to AB: Therefore the ſquare of CD will be to the rectangle contained by CD, GH, that is, twice the triangle

angle GCH, or GEH, as BC to AB. Therefore twice the triangle GEH will be the ſpace to which the ſquare of CD has the ſame ratio that BC has to AB. But it is evident, that twice the ſum of the triangles ADG, BDH is equal to twice the ſum of the triangles AEB, GEH; therefore the ſquare of AD together with the ſpace to which the ſquare of BD has the ſame ratio that BC has to CA, is equal to the rectangle BAC together with the ſpace to which the ſquare of CD has the ſame ratio that BC has to AB. *Q. E. D.*

Corollary. If from the vertex of any triangle there be drawn a line biſecting the baſe, the ſum of the ſquares of the ſides of the triangle will be equal to twice the ſquare of the line biſecting the baſe together with the ſum of the ſquares of the ſegments of the baſe.

PROPOSITION III.

Theorem I. *Fig.* 4.

Let there be any regular figure ABC *circumſcribed about a circle, and from any point* D *within*

within the figure let there be drawn DE, DF, DG *perpendicular to the ſides of the figure; the ſum of the perpendiculars* DE, DF, DG *will be equal to the multiple of the ſemidiameter of the circle by the number of the ſides of the figure.*

Join DA, DB, DC. The figure ABC will be divided into as many triangles as there are ſides in the figure; and becauſe every one of the triangles is equal to half the rectangle contained by the baſe and the perpendicular drawn from the vertex to the baſe, and all the baſes are equal, becauſe the figure is regular; therefore the ſum of all the triangles will be equal to half the rectangle contained by the ſum of the perpendiculars and one of the ſides of the figure; and therefore twice the figure will be equal to the rectangle contained by the ſum of the perpendiculars and one of the ſides of the figure. But the rectangle contained by the ſemidiameter of the circle and the ſum of the ſides of the figure, is equal to twice the figure: Therefore the rectangle contained by the ſum of the perpendiculars DE, DF, DG and one of the ſides of the figure, is equal to the rectangle

angle contained by the ſemidiameter of the circle and the ſum of the ſides of the figure; and therefore the ſum of the perpendiculars DE, DF, DG will be to the ſemidiameter of the circle, as the ſum of the ſides of the figure to one of the ſides of the figure, that is, as the number of the ſides of the figure to one. Therefore the ſum of the perpendiculars DE, DF, DG is equal to the multiple of the ſemidiameter of the circle by the number of the ſides of the figure. *Q. E. D.*

LEMMA I. *Fig.* 5.

Let there be any circle ABC, *and let* AD *be a tangent to the circle in the point* A; *from the point* A *let there be drawn* AB *to any point* B *in the circle, and let* BD *be perpendicular to* AD; *the ſquare of* AB *will be equal to the rectangle contained by* BD *and the diameter.*

Let AC be the diameter of the circle, and join BC. Becauſe the angles ACB, BAD are [32. 3.] equal, and the angles ABC, ADB likewiſe equal, becauſe both right, the triangles ABC, ADB will be ſimilar; therefore AC will be to AB as AB to BD: Therefore the ſquare of AB is equal to the rectangle contained by BD, AC. *Q. E. D.*

PROPOSITION IV.

THEOREM II. *Fig.* 6. 7.

Let the circumference of a circle be divided into any number of equal parts in the points A, B, C, *&c. and from the points* A, B, C, *&c. let there be drawn right lines to any point* D; *the ſum of the ſquares of* AD, BD, CD, *&c. will be equal to the multiple of the ſquare of the line drawn from the centre of the circle to the point* D *by the number of the points* A, B, C, *&c. together with the multiple of the ſquare of the ſemidiameter by the ſame number.*

1. When the point D [*Fig.* 6.] is in the circumference of the circle, it is to be ſhewn, that the ſum of the ſquares of AD, BD, CD, *&c.* is equal to twice the multiple of the

ſquare of the ſemidiameter by the number of the points A, B, C, &c.

Let there be a regular figure circumſcribed about the circle, touching the circle in the points A, B, C, &c. and draw DE, DF, DG perpendicular to the ſides of the figure. Becauſe the ſquare of AD is [*Lem.* 1.] equal to the rectangle contained by DE and the diameter, and likewiſe the ſquare of BD equal to the rectangle contained by DF and the diameter, and ſo on; it is evident, that the ſum of the ſquares of AD, BD, CD, &c. will be equal to the rectangle contained by the ſum of the perpendiculars DE, DF, DG, &c. and the diameter. But becauſe [3.] the ſum of the perpendiculars DE, DF, DG, &c. is equal to the multiple of the ſemidiameter by the number of the ſides of the circumſcribed figure, that is, by the number of the points A, B, C, &c. the rectangle contained by the ſum of the perpendiculars DE, DF, DG, &c. and the diameter, will be equal to twice the multiple of the ſquare of the ſemidiameter by the number of the points A, B, C, &c. Therefore the ſum of the ſquares of AD, BD, CD, &c. will be equal to twice the multiple of the ſquare of the ſemidiameter by the number of the points A, B, C, *&c.* *Q. E. D.*

2. When the point D [*Fig.* 7.] is not in the circumference of the circle, it is to be ſhewn, that the ſum of the ſquares of AD, BD, CD, *&c.* is equal to the multiple of the ſquare of the line drawn from the centre of the circle to the point D by the number of the points A, B, C, *&c.* together with the multiple of the ſquare of the ſemidiameter by the ſame number.

Let E be the centre of the circle, and join DE; let DE meet the circle in the point F on the other ſide of the centre E, and join AE, BE, CE, *&c.* AF, BF, CF, *&c.* The ſquare of AD together with the ſpace to which the ſquare of AF has the ſame ratio that EF has to ED, will [2.] be equal to the rectangle EDF together with the ſpace to which the ſquare of AE, or EF, has the ſame ratio that EF has to FD, that is, together with the rectangle EFD: And therefore the ſquare of AD together with the ſpace to which the ſquare of AF has the ſame ratio that EF has to ED, will be equal to the ſquare of DF. The ſame way it is ſhown, that the ſquare of

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Plate I

Fig. 1

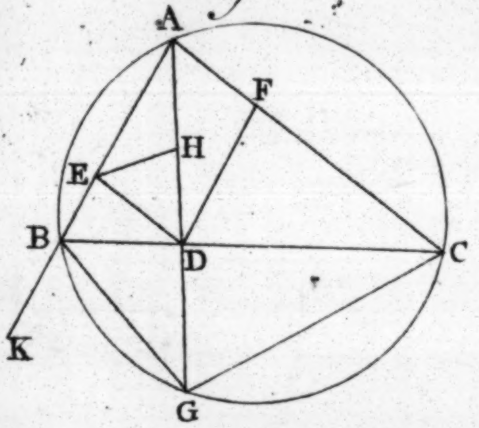


Fig. 2

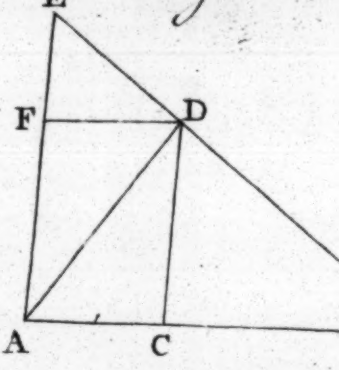


Fig. 4

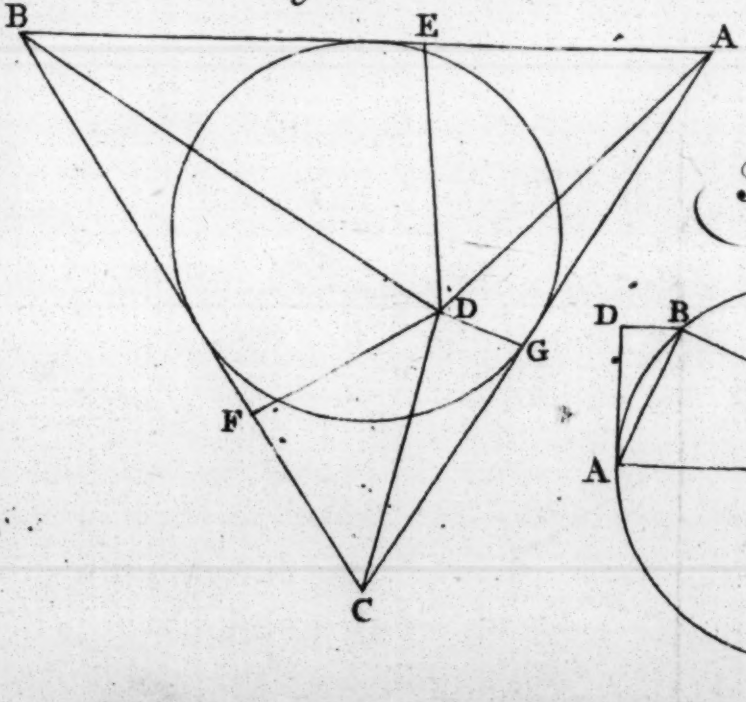
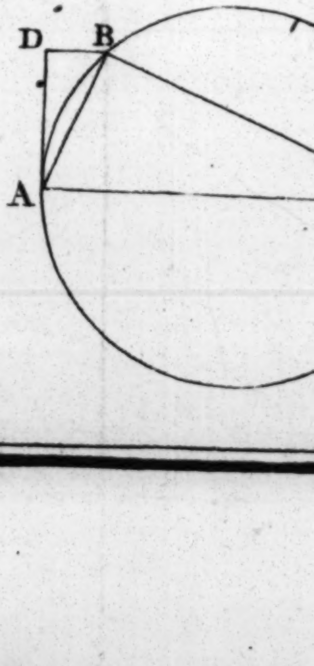


Fig. 5



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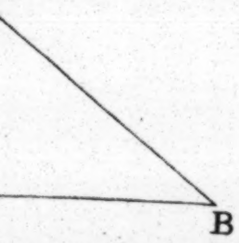


Fig. 3

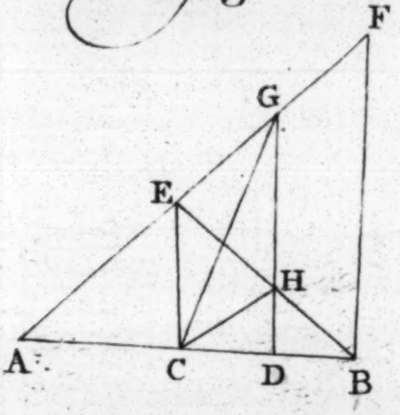
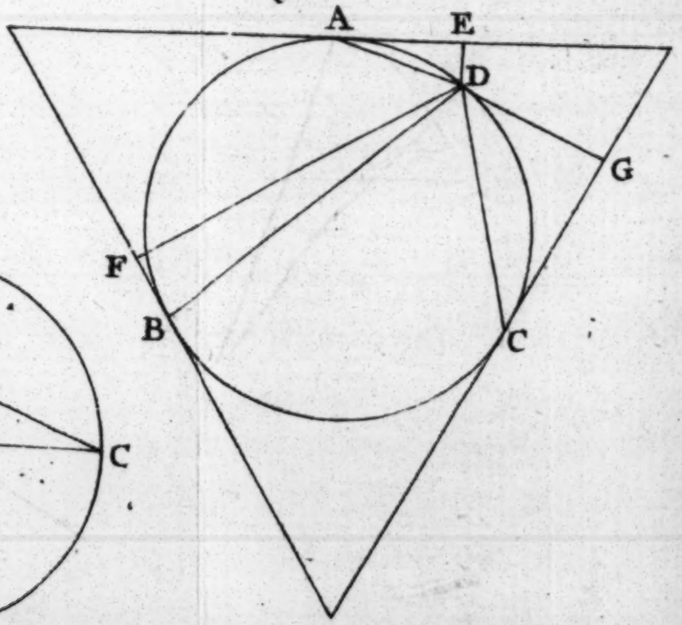


Fig. 5

Fig. 6



BD together with the ſpace to which the ſquare of BF has the ſame ratio that EF has to ED, is equal to the ſquare of DF, and ſo on: Therefore the ſum of the ſquares of AD, BD, CD, *&c.* together with the ſpace to which the ſum of the ſquares of AF, BF, CF, *&c.* has the ſame ratio that EF has to ED, will be equal to the multiple of the ſquare of DF by the number of the points A, B, C, *&c.* But becauſe the ſum of the ſquares of AF, BF, CF, *&c.* is equal [by the firſt part of this] to twice the multiple of the ſquare of EF by the number of the points A, B, C, *&c.* the ſpace to which the ſum of the ſquares of AF, BF, CF, *&c.* has the ſame ratio that EF has to ED, will be equal to twice the multiple of the rectangle FED by the number of the points A, B, C, *&c.*: Therefore the ſum of the ſquares of AD, BD, CD, *&c.* together with twice the multiple of the rectangle FED by the number of the points A, B, C, *&c.* is equal to the multiple of the ſquare of DF by the ſame number: And therefore the ſum of the ſquares of AD, BD, CD, *&c.* is equal to the multiple of the ſum of the ſquares of DE, EF by the number

ber of the points A, B, C, *&c.* *Q. E. D.*

Cor. I. Let there be two circles having the ſame centre, and let the circumference of one of the circles be divided into any number of equal parts, and from the points of diviſion let there be drawn right lines to any point in the circumference of the other, the ſum of the ſquares of theſe lines will always be the ſame.

Cor. II. Let there be two regular figures inſcribed in a circle, and from all the angles of both figures let there be drawn right lines to any point, the ſum of the ſquares of the lines drawn from the angles of the one, will be to the ſum of the ſquares of the lines drawn from the angles of the other, as the number of the ſides of the one to the number of the ſides of the other.

LEMMA II. *Fig.* 8. 9.

Let there be any number of right lines AB, AC, AD, AE, &c. *interſecting each other in the point* A, *and making all the angles about the point* A *equal; let there be any circle paſſing through the point* A; *the circumference*

cumference of the circle will be divided by the lines interſecting each other in the point A *into as many equal parts as there are lines.*

1. When the circle does not touch any of the lines interſecting each other in the point A. [*Fig.* 8.]

Let AB, AC, AD, AE, *&c.* meet the circle in B, C, D, E, *&c.* Becauſe the angles BAC, CAD, DAE, *&c.* are equal, the ſegments BC, CD, DE, *&c.* will be equal. Let BE be the ſegment in which the point A is; draw BD, ED to any point D in the circle; the angle BDE will be equal to the angle adjacent to the angle BAE, that is, to the angle BAF, or BAC; therefore the ſegment BAE is equal to the ſegment BC.

2. When the circle touches one of the lines interſecting each other in the point A. [*Fig.* 9.]

Let it touch AB, and let AC, AD, AE meet the circle in C, D, E. Becauſe the angle CAD is equal to the angle DAE, CD will be equal to DE, *&c.* Join CD; and becauſe the angle ADC is equal to the angle CAB, that is, to the angle CAD, or DAE, the ſegment AC will be equal to the ſegment CD, or DE. The ſame way it is ſhown, that the ſegment AE is equal to the ſegment DE, or DC. Therefore the Lemma is evident. *Q. E. D.*

PROPOSITION V.

THEOREM III. *Fig.* 10. 11.

Let there be any regular figure circumſcribed about a circle, and from any point let there be drawn perpendiculars to the ſides of the figure, and likewiſe a right line to the centre of the circle; twice the ſum of the ſquares of the perpendiculars to the ſides of the figure, will be equal to the multiple of the ſquare of the line drawn to the centre by the number of the ſides of the figure, together with twice the multiple of the ſquare of the ſemidiameter by the ſame number.

1. When the number of the ſides of the figure circumſcribed about the circle is even. [*Fig.* 10.]

Let ABCDEF, *&c.* be any regular figure of an even number of ſides circumſcribed about a circle, and from any point G let there be drawn GH, GK, GL, GM, GN, GO perpendicular to the ſides of the figure, and let *a* be the centre of the circle, and join G*a*; twice the ſum of the ſquares of GH, GK, GL, GM, GN, GO, *&c.* will be equal to the multiple of the ſquare of G*a* by the number of the ſides of the figure, together with twice the multiple of the ſquare of the ſemidiameter of the circle by the ſame number.

Let the circumſcribed figure touch the circle in P, Q, R, S, T, V, *&c.* and join GP, GQ, GR, GS, GT, GV, *&c.* join *a*P, *a*Q, *a*R, *&c.* and draw GX, GY, GZ, *&c.* perpendicular to *a*P, *a*Q, *a*R, *&c.*

Becauſe the number of the ſides of the circumſcribed figure is even, it is plain, that *a*P, *a*Q, *a*R, *&c.* will paſs through the oppoſite points of contact, that is, through the points S, T, V; and therefore the number of lines interſecting each other in the point *a* will be half the number of the ſides of the figure, and all the angles round the point *a* will be equal. Becauſe the ſum of the ſquares of GH, GX is equal to the ſquare of GP, and the ſum of the ſquares of GK, GY equal to the ſquare of GQ, and ſo on; it is evident, that the ſum of the ſquares of GH, GK, GL, GM, GN, GO, *&c.* together with twice the ſum of the ſquares of GX, GY, GZ, *&c.* is equal to the ſum of the ſquares of GP, GQ, GR, GS, GT, GV, *&c*, that is, [4.], equal to the multiple of the ſquare of G*a* by the number of the ſides of the figure together with the multiple of the ſquare of the ſemidiameter of the circle by the ſame number: Therefore twice the ſum of the ſquares of GH, GK, GL, GM, GN, GO, *&c.* together with four times the ſum of the ſquares of GX, GY, GZ, *&c.* will be equal to twice the multiple of the ſquare of G*a* by the number of the ſides of the figure together with twice the multiple of the ſquare of the ſemidiameter of the circle by the ſame number. Again, Becauſe the angles GX*a*, GZ*a*, GY*a* are right, the points X, Y, Z will be in the circumference of the circle whoſe diameter is G*a*; and becauſe the circle paſſes through the point *a*, the circumference will be divided into equal parts in the points X, Y, Z as many in number as there are right lines *a*P, *a*Q, *a*R, *&c.* [*Lem.* 2.]: Biſect G*a* in *b*; the ſum of the ſquares of GX, GY, GZ, &c. will [4.] be equal to twice the multiple of the ſquare of Gb by the number of the lines aP, aQ, aR, &c. that is, (becauſe the number of the lines aP, aQ, aR, &c. is equal to half the number of the ſides of the figure), equal to the multiple of the ſquare of Gb by the number of the ſides of the circumſcribed figure; and therefore four times the ſum of the ſquares of GX, GY, GZ, &c. will be equal to the multiple of the ſquare of aG by the number of the ſides of the figure. Therefore twice the ſum of the ſquares of GH, GK, GL, GM, GN, GO, &c. together with the multiple of the ſquare of Ga by the number of the ſides of the circumſcribed figure, will be equal to twice the multiple of the ſquare of Ga by the number of the ſides of the figure together with twice the multiple of the ſquare of the ſemidiameter by the ſame number: And therefore the ſum of the ſquares of GH, GK, GL, GM, GN, GO, &c. will be equal to the multiple of the ſquare of Ga by the number of the ſides of the figure together with twice the multiple of the ſquare of the ſemidiameter by the ſame number.

2. When the number of the ſides of the fi-

gure circumſcribed about the circle is odd. [*Fig.* 11.]

Let ABCDE, *&c.* be any regular figure of an odd number of ſides circumſcribed about a circle, and from any point F let there be drawn FG, FH, FK, FL, FM, *&c.* perpendicular to the ſides of the figure, and let *a* be the centre of the circle, and join F*a*; twice the ſum of the ſquares of FG, FH, FK, FL, FM, *&c.* will be equal to the multiple of the ſquare of F*a* by the number of the ſides of the figure together with twice the multiple of the ſquare of the ſemidiameter of the circle by the ſame number.

Let the circumſcribed figure touch the circle in the points N, O, P, Q, R, *&c.* and join FN, FO, FP, FQ, FR, *&c.* join *a*N, *a*O, *a*P, *a*Q, *a*R, *&c.* and draw FS, FT, FV, FX, FY, *&c.* perpendicular to *a*N, *a*O, *a*P, *a*Q, *a*R, *&c.* Becauſe the ſum of the ſquares of FG, FS is equal to the ſquare of FN, and the ſum of the ſquares of FH, FT equal to the ſquare of FO, and ſo on; it is evident, that the ſum of the ſquares of FG, FH, FK, FL, FM, *&c.* together with the ſum of the ſquares of FS, FT, FV, FX, FY, *&c.* is equal to the ſum of the ſquares of FN, FO, FP, FQ, FR, &c. that is, [4.] equal to the multiple of the ſquare of F*a* by the number of the ſides of the figure together with the multiple of the ſquare of the ſemidiameter by the ſame number; therefore twice the ſum of the ſquares of FG, FH, FK, FL, FM, &c. together with twice the ſum of the ſquares of FS, FT, FV, FX, FY, &c. is equal to twice the multiple of the ſquare of F*a* by the number of the ſides of the figure together with twice the multiple of the ſquare of the ſemidiameter of the circle by the ſame number. Again, Becauſe the angles FS*a*, FT*a*, FV*a*, FX*a*, FY*a*, &c. are right, the points S, T, V, X, Y, &c. will be in the circumference of the circle whoſe diameter is F*a*; and becauſe the circle paſſes through the point *a*, and the lines *a*N, *a*O, *a*P, *a*Q, *a*R, &c. make all the angles round the point *a* equal, the circumference of the circle will be divided into equal parts in the points S, T, V, X, Y, &c. as many in number as there are right lines *a*N, *a*O, *a*P, *a*Q, *a*R, &c. [*Lem.* 2.]. Biſect F*a* in *b*; the ſum of the ſquares of FS, FT, FV, FX, FY, &c, will be equal to twice the multiple of the ſquare of F*b* by the number

ber of the lines *a*N, *a*O, *a*P, *a*Q, *a*R, *&c.* [4.] that is, by the number of the ſides of the circumſcribed figure; and therefore twice the ſum of the ſquares of FS, FT, FV, FX, FY, *&c.* will be equal to the multiple of the ſquare of F*a* by the number of the ſides of the figure. Therefore twice the ſum of the ſquares of FG, FH, FK, FL, FM, *&c.* together with the multiple of the ſquare of F*a* by the number of the ſides of the figure, will be equal to twice the multiple of the ſquare of F*a* by the number of the ſides of the figure together with twice the multiple of the ſquare of the ſemidiameter of the circle by the ſame number: And therefore twice the ſum of the ſquares of FG, FH, FK, FL, FM, *&c.* will be equal to the multiple of the ſquare of F*a* by the number of the ſides of the figure together with twice the multiple of the ſquare of the ſemidiameter of the circle by the ſame number. *Q. E. D.*

Cor. I. Let there be any regular figure circumſcribed about a circle, and from any point in the circumference of the circle let there be drawn perpendiculars to the ſides of the figure; twice the ſum of the ſquares of the perpendiculars

diculars will be equal to thrice the multiple of the ſquare of the ſemidiameter of the circle by the number of the ſides of the figure.

Cor. II. Let there be two circles having the ſame centre, and from any point in the circumference of the one let there be drawn perpendiculars to the ſides of any regular figure circumſcribed about the other; the ſum of the ſquares of theſe perpendiculars will always be the ſame.

Cor. III. Let there be two regular figures circumſcribed about a circle, and from any point let there be drawn perpendiculars to the ſides of both figures; the ſum of the ſquares of the perpendiculars drawn to the ſides of the one, will be to the ſum of the ſquares of the perpendiculars drawn to the ſides of the other, as the number of the ſides of the one to the number of the ſides of the other.

PROPOSITION VI. *Fig.* 12. 13.

Let A, B *be two points in the ſemidiameter of a circle whoſe centre is* C, *and let the rectangle* ACB *be equal to the ſquare of the ſemidiameter*; *biſect* AB *in* D, *and draw*

DE *perpendicular to* AB; *from the point* A *draw* AF *to any point* F *in the circle, and draw* FE *perpendicular to* DE; *the ſquare of* AF *will be equal to twice the rectangle contained by* AC, FE.

Let CG be equal to AC, and join GF; let EF meet the circle in H, and join AH, GH, AE, CE, CF, CH; and let CE meet the circle in K, L.

The ſquare of CD is equal to the rectangle ACB together with the ſquare of AD, that is, equal to the ſquare of the ſemidiameter together with the ſquare of AD. Add the ſquare of DE to both; and the ſquare of CE will be equal to the ſquare of the ſemidiameter together with the ſquare of AE. Take away the ſquare of the ſemidiameter from both; and the ſquare of AE will be equal to the rectangle KEL, that is, equal to the rectangle FEH: And therefore FE is to AE, as AE to EH: Therefore the triangles AEF, AHE are ſimilar, and the angle EAF will be equal to the angle AHE, that is, equal to the angle HAG. Again, Becauſe the angle ACF is equal to the angle CFH, that is, equal to the angle CHF, or GCH, the angle ACH will be equal to the angle GCF; and becauſe AC, CH are equal to GC, CF, the triangles ACH, GCF will be every way equal, and the angle CGF will be equal to the angle HAG, that is, equal to the angle EAF; and becauſe the angles EFA, FAG are equal, the triangles AEF, FAG will be ſimilar; and therefore EF will be to AF, as AF to AG: Therefore the ſquare of AF is equal to the rectangle contained by EF, AG, that is, equal to twice the rectangle contained by FE, AC. *Q. E. D.*

PROPOSITION. VII.

THEOREM IV. *Fig.* 14. 15.

Let there be any circle whoſe centre is A, *and let* BCD *be a ſegment of the circle, and* BD *the chord of the ſegment; about the ſegment let there be any equilateral figure circumſcribed touching the circle in the points* E, F, G, &c. *and let the two ſides of the figure next to* BD *meet* BD *in* H, K; *biſect the ſegment* BCD *in* F, *and join* AF; *in* AF *take the point* L *on the ſame ſide the centre*

centre A *with the point* F, *and let the ſum of the ſides of the figure circumſcribed about the ſegment be to* HK *as the ſemidiameter to* AL; *draw* LM *perpendicular to* AL *meeting the circle in* M. *If from the points* E, F, G, &c. *the points of contact of the circumſcribed figure, and the point* L, *there be drawn right lines to any point* N, *the ſum of the ſquares of* EN, FN, GN, &c. *will be equal to the multiple of the ſum of the ſquares of* LM, LN *by the number of the ſides of the figure.*

1. When the point N is in the circumference of the circle. [*Fig.* 14.]

In AF take the point O, and let the rectangle LAO be equal to the ſquare of the ſemidiameter of the circle, and let OP be perpendicular to AF; draw NP perpendicular to OP; biſect LO in Q, and let QR parallel to OP meet NP in R; let NP, AO meet BD in S, T, and join AH, AK, NH, NK; and join likewiſe AM; and draw NV, NX, NY, *&c.* perpendicular to the ſides of the figure meeting the ſides of the figure in V, X, Y, &c.

Becauſe

Plate II p

Fig. 7

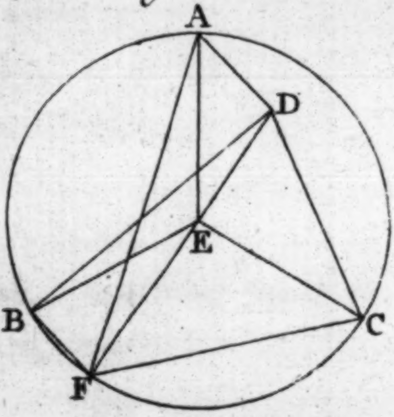


Fig. 8

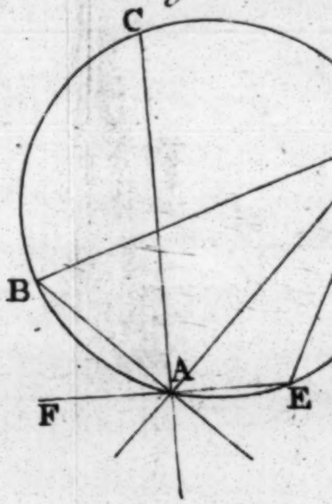


Fig. 10

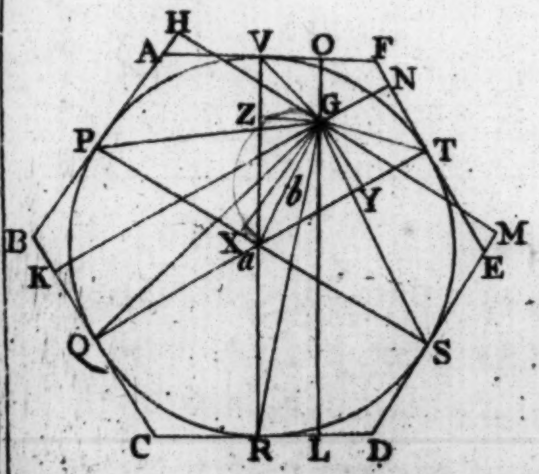
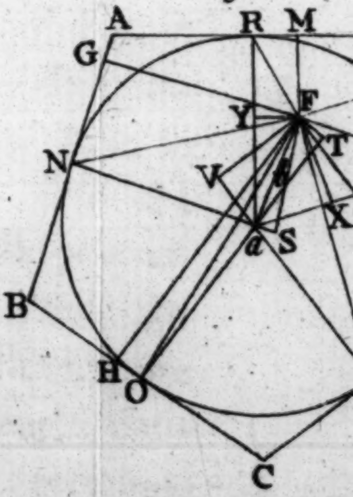


Fig. 11



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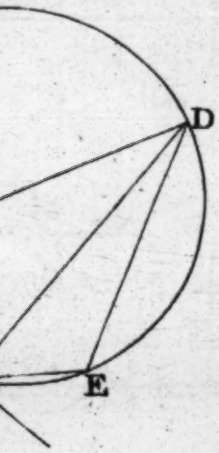
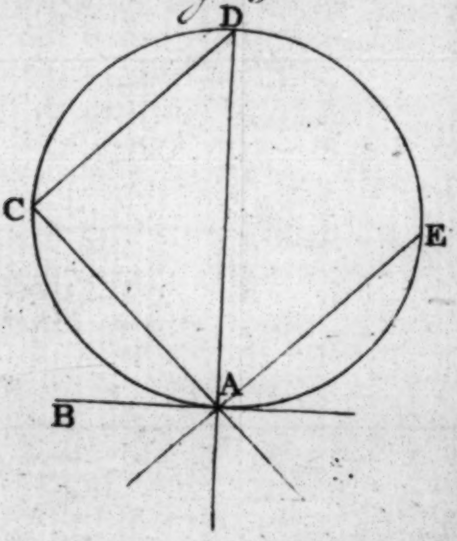


Fig. 9



ig. 11

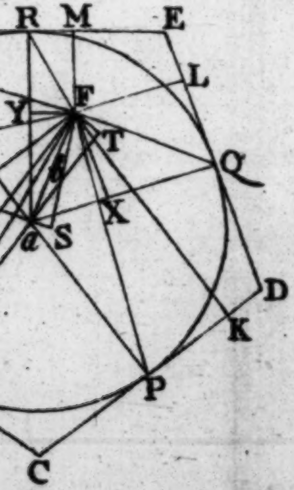
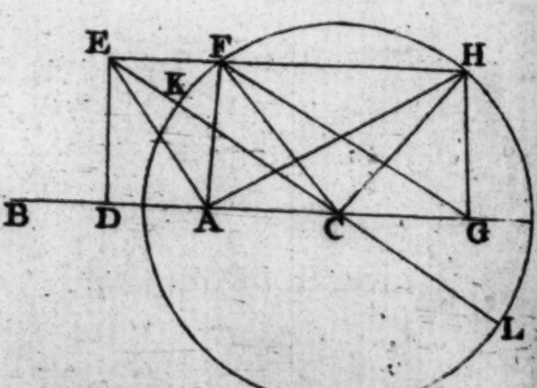


Fig. 12



Becauſe the rectangle LAO is equal to the ſquare of the ſemidiameter of the circle, that is, equal to the ſquare of AF; AO will be to AF, as AF to AL, that is, as the ſum of the ſides of the figure circumſcribed about the ſegment to HK; and therefore the rectangle contained by AO, HK will be equal to the rectangle contained by AF and the ſum of the ſides of the figure, that is, will be equal to twice the figure AHEFGKA. And becauſe the rectangle contained by AT, HK is equal to twice the triangle AHK, the rectangle contained by OT, HK will be equal to twice the figure HEFGKH, that is, the rectangle contained by PS, HK will be equal to twice the figure HEFGKH. Again, Becauſe the rectangle contained by NS, HK is equal to twice the triangle NHK, the rectangle contained by NP, HK will be equal to twice the figure NHEFGKN. But the rectangle contained by the ſum of the perpendiculars NV, NX, NY, &c. and one of the ſides of the figure, is equal to twice the figure NHEFGKN; therefore the rectangle contained by NP, HK is equal to the rectangle contained by the ſum of NV, NX, NY, &c.

and one of the ſides of the figure: And therefore NP will be to one of the ſides of the figure, as the ſum of the perpendiculars NV, NX, NY to HK, that is, the multiple of NP by the number of the ſides of the figure, will be to the ſum of the ſides of the figure, as the ſum of the perpendiculars NV, NX, NY, &c. to HK: Therefore the multiple of NP by the number of the ſides of the figure, will be to the ſum of the perpendiculars NV, NX, NY, as the ſum of the ſides of the figure to HK, that is, as AF to AL, or twice AF to twice AL: Therefore twice the multiple of the rectangle contained by NP, AL by the number of the ſides of the figure, is equal to the rectangle contained by the ſum of the perpendiculars NV, NX, NY, &c. and twice AF. But becauſe [*Lem.* 1.] the ſquare of NE is equal to the rectangle contained by NV and twice AF, and the ſquare of NF equal to the rectangle contained by NX and twice AF, and the ſquare of NG equal to the rectangle contained by NY and twice AF, and ſo on; the ſum of the ſquares of NE, NF, NG, &c. will be equal to the rectangle contained by the ſum of the perpendiculars

NV, NX, NY, *&c.* and twice AF, that is, will be equal to twice the multiple of the rectangle contained by NP, AL by the number of the ſides of the figure. Again, Becauſe the rectangle OAL is equal to the ſquare of AM, the rectangle OLA will be equal to the ſquare of LM, that is, twice the rectangle contained by PR, AL will be equal to the ſquare of LM. And becauſe [6.] twice the rectangle contained by NR, AL is equal to the ſquare of LN, twice the rectangle contained by NP, AL will be equal to the ſum of the ſquares of LM, LN: Therefore twice the multiple of the rectangle contained by NP, AL by the number of the ſides of the figure, will be equal to the multiple of the ſum of the ſquares of LM, LN by the ſame number: And therefore the ſum of the ſquares of NE, NF, NG, *&c.* will be equal to the multiple of the ſum of the ſquares of LM, LN by the number of the ſides of the figure. *Q. E. D.*

2. When the point N is not in the circumference of the circle. [*Fig.* 15.]

Join NA, and let NA meet the circle in the point O on the other ſide the centre A with the point N; and join EO, FO, GO, *&c.* LO; join likewiſe EA, FA, GA, *&c.* and join AM; let LO meet the circle in P, and draw NQ parallel to AL meeting OL in Q.

Becauſe [2.] the ſquare of EN together with the ſpace to which the ſquare of EO has the ſame ratio that OA has to AN, is equal to the rectangle ANO together with the ſpace to which the ſquare of AE has the ſame ratio that AO has to ON, and the ſquare of AE is equal to the ſum of the ſquares of AL, LM; the ſquare of EN together with the ſpace to which the ſquare of EO has the ſame ratio that OA has to AN, will be equal to the rectangle ANO together with the ſpace to which the ſquare of AL has the ſame ratio that AO has to ON together with the ſpace to which the ſquare of LM has the ſame ratio. But [2.] the rectangle ANO together with the ſpace to which the ſquare of AL has the ſame ratio that AO has to NO, is equal to the ſquare of NL together with the ſpace to which the ſquare of LO has the ſame ratio that OA has to AN. Therefore the ſquare of EN together with the ſpace to which the ſquare of EO has the ſame ratio that OA has to AN, is equal to the ſquare of NL together with the ſpace to which the ſquare of LO has the ſame ratio that OA has to AN together with the ſpace to which the ſquare of LM has the ſame ratio that OA has to ON. Becauſe the ſquare of LO is to the rectangle OLQ as OL to LQ, that is, as OA to AN, the rectangle OLQ will be the ſpace to which the ſquare of OL has the ſame ratio that OA has to AN. And becauſe the rectangle OLP is to the rectangle contained by LP, OQ as OL to OQ, that is, as OA to ON, and the ſquare of LM is equal to the rectangle OLP, the ſquare of LM will be to the rectangle contained by LP, OQ as OA to ON: Therefore the rectangle contained by LP, OQ will be the ſpace to which the ſquare of LM has the ſame ratio that OA has to ON. And therefore the ſquare of EN together with the ſpace to which the ſquare of EO has the ſame ratio that OA has to AN, is equal to the ſquare of NL together with the rectangle OLQ together with the rectangle contained by LP, OQ. But becauſe the rectangle contained by LP, OQ is equal to the rectangle OLP together with the rectangle contained by LP, LQ; therefore the rectangle OLQ together with the rectangle contained by LP, OQ is equal to the rectangle OLP together with the rectangle contained by OP, LQ, that is, equal to the ſquare of LM together with the rectangle contained by OP, LQ. Therefore the ſquare of NE together with the ſpace to which the ſquare of EO has the ſame ratio that OA has to AN, is equal to the ſum of the ſquares of LM, LN together with the rectangle contained by OP, LQ. The ſame way it is ſhewn, that the ſquare of FN together with the ſpace to which the ſquare of FO has the ſame ratio that OA has to AN, is equal to the ſum of the ſquares of LM, LN together with the rectangle contained by OP, LQ; and likewiſe, that the ſquare of GN together with the ſpace to which the ſquare of GO has the ſame ratio that OA has to AN, is equal to the ſum of the ſquares of LM, LN together with the rectangle contained by OP, LQ; and ſo on. Therefore the ſum of the ſquares of EN, FN, GN, *&c.* together with the ſpace to which the ſum of the ſquares of EO, FO, GO, *&c.* has the ſame ratio that OA has to

AN, is equal to the multiple of the ſum of the ſquares of LM, LN by the number of the ſides of the figure together with the multiple of the rectangle contained by OP, LQ by the ſame number.

Again, Becauſe the ſum of the ſquares of EO, FO, GO, *&c.* is (by the firſt part of this) equal to the multiple of the ſum of the ſquares of LM, LO by the number of the ſides of the figure, and the ſquare of LM is equal to the rectangle OLP; the ſum of the ſquares of EO, FO, GO, *&c.* will be equal to the multiple of the rectangle LOP by the number of the ſides of the figure. And becauſe OA is to AN as OL to LQ, that is, as the rectangle LOP to the rectangle contained by OP, LQ, that is, as the multiple of the rectangle LOP by the number of the ſides of the figure to the multiple of the rectangle contained by OP, LQ by the ſame number, and the ſum of the ſquares of EO, FO, GO, *&c.* is equal to the multiple of the rectangle LOP by the number of the ſides of the figure; the ſum of the ſquares of EO, FO, GO, *&c.* will be to the multiple of the rectangle contained by OP, LQ as OA to AN. And therefore the multiple of the rectangle contained by OP, LQ by the number of the ſides of the figure, will be the ſpace to which the ſum of the ſquares of EO, FO, GO, &c. has the ſame ratio that OA has to AN: Therefore the ſum of the ſquares of EN, FN, GN, &c. together with the multiple of the rectangle contained by OP, LQ by the number of the ſides of the figure, is equal to the multiple of the ſum of the ſquares of LM, LN by the number of the ſides of the figure together with the multiple of the rectangle contained by OP, LQ by the ſame number. And therefore the ſum of the ſquares of EN, FN, GN, &c. will be equal to the multiple of the ſum of the ſquares of LM, LN by the number of the ſides of the figure. *Q. E. D.*

PROPOSITION VIII.

THEOREM V. *Fig.* 16.

Let there be any circle whoſe centre is A, *and let* BCD *be a ſemicircle, and* BD *the diameter of the circle; about the ſemicircle let there*

there be any regular figure deſcribed, and let the ſides of the figure next to BD *meet* BD *in* E, F; *biſect the ſemicircle in* G, *and join* AG; *and in* AG *take the point* H *on the ſame ſide the centre* A *with the point* G, *and let* AG *be to* AH *as the ſum of the ſides of the figure to* EF; *and let the rectangle* HAK *be equal to the ſquare of the ſemidiameter, and let* HL *be equal to* AH: *If from any point* M *there be drawn* MN, MO, MP, &c. *perpendicular to the ſides of the figure circumſcribed about the ſemicircle, and likewiſe there be drawn* ML *to the point* L; *twice the ſum of the ſquares of the perpendiculars* MN, MO, MP, &c. *will be equal to the multiple of the ſquare of* ML *by the number of the ſides of the figure together with the multiple of the rectangle* KLA *by the ſame number.*

Let the figure touch the ſemicircle in the points Q, R, S, *&c.* and join AQ, AR, AS, *&c.*; draw MT, MV, MX, *&c.* perpendicular to AQ, AR, AS; join MA, MH, and draw HY perpendicular to AH meeting

the

the circle in Y, and join MQ, MR, MS, *&c.*

Becauſe [7.] the ſum of the ſquares of MQ, MR, MS, *&c.* is equal to the multiple of the ſum of the ſquares of HM, HY by the number of the ſides of the figure circumſcribed about the ſemicircle, twice the ſum of the ſquares of MQ, MR, MS, *&c.* will be equal to twice the multiple of the ſum of the ſquares of HM, HY by the number of the ſides of the figure: Therefore twice the ſum of the ſquares of MQ, MR, MS, *&c.* together with twice the multiple of the ſquare of AH by the number of the ſides of the figure, is equal to twice the multiple of the ſum of the ſquares of HM, HA by the number of the ſides of the figure together with twice the multiple of the ſquare of HY by the ſame number. And becauſe twice the ſum of the ſquares of HM, HA is equal to the ſum of the ſquares of ML, MA, twice the multiple of the ſum of the ſquares of HM, HA by the number of the ſides of the figure, will be equal to the multiple of the ſum of the ſquares of ML, MA by the ſame number: Therefore twice the ſum of the ſquares of MQ, MR, MS, *&c.* together with twice the multiple of the ſquare of AH by the number of the ſides of the figure, is equal to the multiple of the ſum of the ſquares of ML, MA by the ſame number. But becauſe twice the ſum of the ſquares of MN, MO, MP, *&c.* together with twice the ſum of the ſquares of MT, MV, MX, *&c.* is equal to twice the ſum of the ſquares of MQ, MR, MS, *&c.*; therefore twice the ſum of the ſquares of MN, MO, MP together with twice the ſum of the ſquares of MT, MV, MX, *&c.* is equal to the multiple of the ſum of the ſquares of ML, MA by the number of the ſides of the figure together with twice the multiple of the ſquare of HY by the ſame number.

Again, Becauſe the angles MTA, MVA, MXA are right, the points T, V, X will be in the circumference of the circle whoſe diameter is AM; and becauſe AQ, AR, AS, *&c.* make all the angles about the point A equal, the circumference of this circle will be divided into equal parts in the points T, V, X, *&c.* [*Lem.* 2.] as many in number as there are lines AQ, AR, AS, *&c.* that is, into as many equal parts as there are ſides in the circumſcribed figure: Therefore twice the ſum of the ſquares of MT, MV, MX, *&c.* will be equal to the multiple of the ſquare of MA by the number of the ſides of the figure: Therefore twice the ſum of the ſquares of MN, MO, MP, *&c.* together with the multiple of the ſquare of MA by the number of the ſides of the figure together with twice the multiple of the ſquare of AH by the ſame number, is equal to the multiple of the ſum of the ſquares of ML, MA by the number of the ſides of the figure together with twice the multiple of the ſquare of HY by the ſame number: And therefore twice the ſum of the ſquares of MN, MO, MP, *&c.* together with twice the multiple of the ſquare of AH by the number of the ſides of the figure, is equal to the multiple of the ſquare of ML by the number of the ſides of the figure together with twice the multiple of the ſquare of HY by the ſame number.

Again, Becauſe the rectangle HAK is equal to the ſquare of the ſemidiameter of the circle, that is, equal to the ſum of the ſquares of AH, HY; the rectangle KHA, that is, the rectangle KHL, will be equal to the ſquare of HY: And therefore twice the multiple of the rectangle KHL by the number of the ſides of the figure, will be equal to twice the multiple of the ſquare of HY by the ſame number: Therefore twice the ſum of the ſquares of MN, MO, MP, *&c.* together with twice the multiple of the ſquare of AH or HL by the number of the ſides of the figure, is equal to the multiple of the ſquare of ML by the number of the ſides of the figure together with twice the multiple of the rectangle KHL by the ſame number. Therefore twice the ſum of the ſquares of MN, MO, MP, *&c.* is equal to the multiple of the ſquare of ML by the number of the ſides of the figure together with the multiple of the rectangle KLA by the ſame number. *Q. E. D.*

Cor. Let there be any equilateral figure inſcribed in a ſemicircle; a point is given, ſuch, that if from any point there be drawn perpendiculars to the ſides of the figure, and likewiſe a right line to the given point, twice the ſum of the ſquares of the perpendiculars will be equal to the multiple of the ſquare of the line drawn to the given point by the number of the ſides of the figure together with a given ſpace.

Let ABCD [*Fig.* 17.] be an equilateral figure inſcribed in a ſemicircle; let AD be the diameter, and F the centre; biſect the ſemicircle in G, and join FG; let FH be perpendicular to AB one of the ſides of the figure; in FG take the point K, and let FH be to FK as the ſum of the ſides of the figure ABCD to AD; let KL be equal to FK, and let the rectangle KFM be equal to the ſquare of FH: If from any point N there be drawn NO, NP, NQ, *&c.* perpendicular to AB, BC, CD, *&c.* the ſides of the figure, and likewiſe there be drawn NL to the point L, twice the ſum of the ſquares of NO, NP, NQ, *&c.* will be equal to the multiple of the ſquare of NL by the number of the ſides of the figure together with the multiple of the rectangle MLF by the ſame number.

The ſecond and fourth theorems are but particular caſes of one more general; which is this.

PRO-

PROPOSITION IX.

THEOREM VI.

Let there be any number of given points; a point may be found, ſuch, that if from all the given points there be drawn right lines to the point found, and from all the given points and the point found there be drawn right lines to any point, the ſum of the ſquares of the lines drawn from the given points, will be equal to the ſum of the ſquares of the lines drawn from the given points to the point found together with the multiple by the number of the given points of the ſquare of the line drawn from the point found.

For example: Let the number of the given points be three, and the theorem will be as follows.

Let there be three given points; a point may be found, ſuch, that if from the three given points there be drawn right lines to the point found, and from the three given points and the point found there be drawn right lines to any point, the ſum of the ſquares of the lines drawn from the three given points, will be equal to the ſum of the ſquares of the lines drawn from the three given points to the point found together with thrice the ſquare of the line drawn from the point found.

This theorem may be made more general thus.

PROPOSITION X.

THEOREM VII.

Let there be any number of given points A, B, C, &c. [Fig. 18.] *and let a, b, c,* &c. *be given magnitudes as many in number as there are given points; a point* X *may be found, ſuch, that if from the given points* A, B, C, &c. *there be drawn right lines to the point* X, *and from the given points and the point* X *there be drawn right lines to any point* Y, *the ſquare of* AY, *together with the ſpace to which the ſquare of* BY *has the ſame ratio that a has to b, together with the ſpace to which the ſquare of* CY *has the ſame*

ſame ratio that a has to c, and ſo on; will be equal to the ſquare of AX, *together with the ſpace to which the ſquare of* BX *has the ſame ratio that a has to b, together with the ſpace to which the ſquare of* CX *has the ſame ratio that a has to c, and ſo on, together with the ſpace to which the ſquare of* XY *has the ſame ratio that a has to the ſum of a, b, c,* &c.

Let the number of the given points be three, and let *a*, *b*, *c* be equal to 1, 2, 3; and the theorem will be as follows.

Let A, B, C be three given points; a point X may be found, ſuch, that if from the points A, B, C there be drawn right lines to the point X, and from the points A, B, C and the point X there be drawn right lines to any point Y, the ſquare of AY together with twice the ſquare of BY together with thrice the ſquare of CY, will be equal to the ſquare of AX together with twicc the ſquare of BX together with thrice the ſquare of CX together with ſix times the ſquare of XY.

Cor. I. Let there be any number of circles given by poſition, and about each of the circles

let

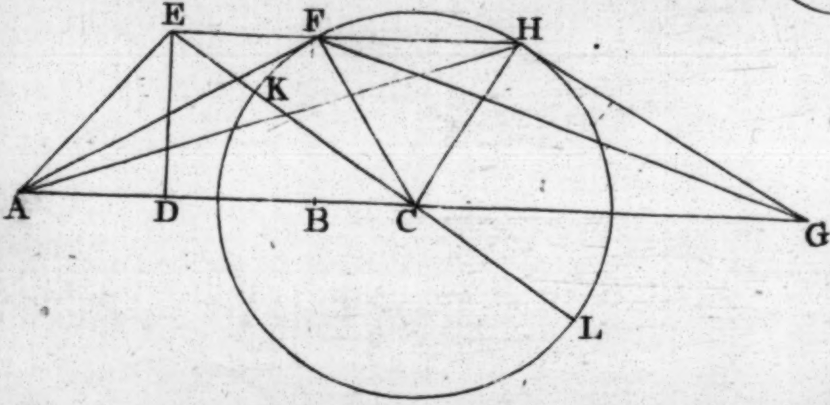
let there be an equilateral figure circumſcribed; a point may be found, ſuch, that if from any point there be drawn perpendiculars to the ſides of all the figures, and likewiſe there be drawn a right line to the point found, twice the ſum of the ſquares of the perpendiculars will be equal to the multiple of the ſquare of the line drawn to the point found by the number of the ſides of all the figures together with a given ſpace.

Suppoſe, for example, two circles to be given by poſition, and about one of the circles let there be an equilateral triangle circumſcribed, and about the other let there be a ſquare circumſcribed; a point may be found, ſuch, that if from any point there be drawn perpendiculars to the ſides of the triangle and the ſquare, and likewiſe there be drawn a right line to the point found, twice the ſum of the ſquares of the perpendiculars will be equal to ſeven times the ſquare of the line drawn to the point found together with a given ſpace.

Cor. II. Let there be any number of ſemicircles given by poſition, and about each of the ſemicircles let there be an equilateral figure circumſcribed; a point may be found,

Plate III

Fig. 13



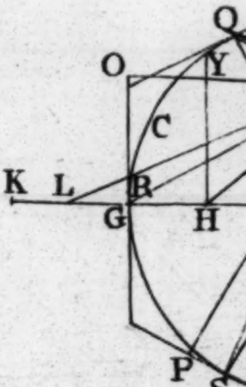


Fig. 15

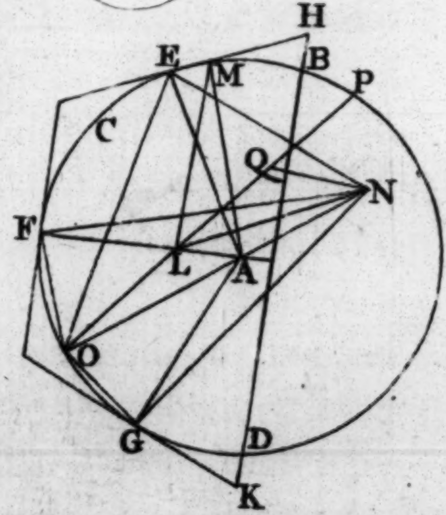
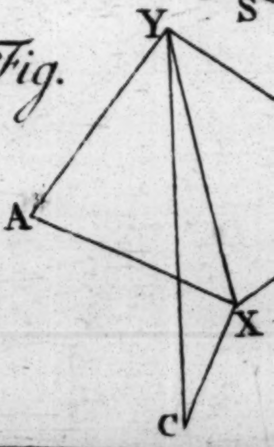


Fig.



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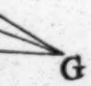


Fig. 14

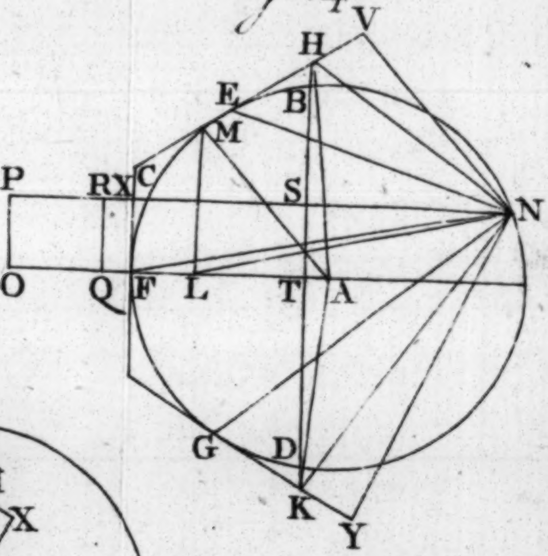
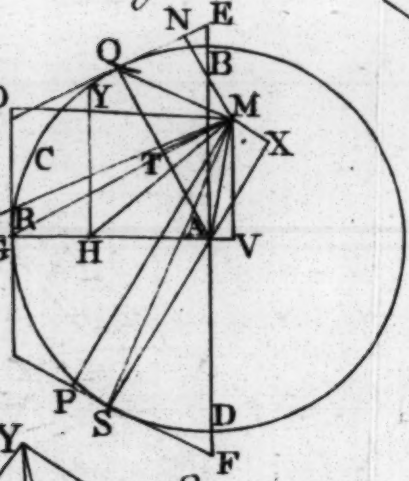


Fig. 16



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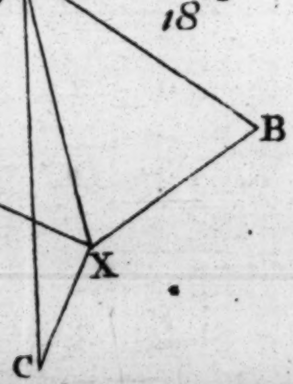
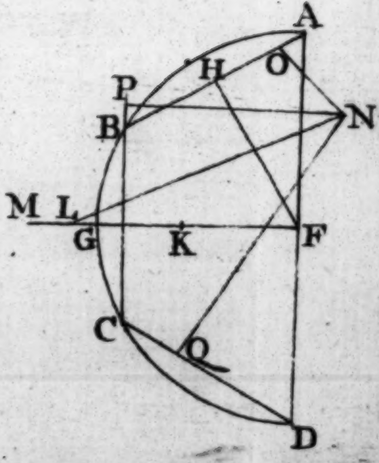


Fig. 17



ſuch, that if from any point there be drawn perpendiculars to the ſides of all the figures, and likewiſe there be drawn a right line to the point found, twice the ſum of the ſquares of the perpendiculars will be equal to the multiple of the ſquare of the line drawn to the point found by the number of the ſides of all the figures together with a given ſpace.

Suppoſe, for example, two ſemicircles to be given by poſition, and about each of the ſemicircles let there be an equilateral figure circumſcribed, and let the number of the ſides of the one be three, and the number of the ſides of the other be four; a point may be found, ſuch, that if from any point there be drawn perpendiculars to the ſides of both the figures, and likewiſe there be drawn a right line to the point found, twice the ſum of the ſquares of the perpendiculars drawn to the ſides of the figures, will be equal to ſeven times the ſquare of the line drawn to the point found together with a given ſpace.

Cor. III. Let there be any number of circles given by poſition, and likewiſe any number of ſemicircles given by poſition; and about each of the circles let there be an equilateral

teral figure circumſcribed, and likewiſe about each of the ſemicircles let there be an equilateral figure circumſcribed: a point may be found, ſuch, that if from any point there be drawn perpendiculars to the ſides of all the figures, and likewiſe a right line to the point found, twice the ſum of the ſquares of the perpendiculars will be equal to the multiple of the ſquare of the line drawn to the point found by the number of the ſides of all the figures.

Suppoſe, for example, two circles to be given by poſition, and likewiſe two ſemicircles to be given by poſition; and about each of the circles let there be an equilateral figure circumſcribed, and likewiſe about each of the ſemicircles let there be an equilateral figure circumſcribed; and let the number of the ſides of the figure circumſcribed about one of the circles be three, and the number of the ſides of the figure circumſcribed about the other be four; and let the number of the ſides of the figure circumſcribed about one of the ſemicircles be three, and the number of the ſides of the figure circumſcribed about the other be five: a point may be found, ſuch, that if from any point there be drawn perpendiculars to the ſides of all the figures, and likewiſe there be drawn a right line to the point found, twice the ſum of the ſquares of the perpendiculars drawn to the ſides of the figures, will be equal to fifteen times the ſquare of the line drawn to the point found together with a given ſpace.

From the two laſt theorems the two following theorems may be eaſily derived.

PROPOSITION. XI.

THEOREM VIII.

Let there be any number of given points, two points may be found, ſuch, that if from all the given points and the two points found there be drawn right lines to any point, twice the ſum of the ſquares of the lines drawn from the given points, will be equal to the multiple by the number of the given points of the ſum of the ſquares of the lines drawn from the two points found.

Let the number of the given points be three, and the theorem will be as follows.

Let there be three given points, two points may be found, ſuch, that if from the three given points and the two points found there be drawn right lines to any point, twice the ſum of the ſquares of the lines drawn from the three given points, will be equal to ſix times the ſum of the ſquares of the lines drawn from the two points found; and ſo on.

PROPOSITION XII.

THEOREM IX.

Let there be any number of given points, and let a, b, c, &c. be given magnitudes, as many in number as there are given points; two points may be found, ſuch, that if from all the given points and the two points found there be drawn right lines to any point, the ſquare of the line drawn from one of the given points, together with the ſpace to which the ſquare of the line drawn from another of the given points has the ſame ratio that a has to b, together with the ſpace to which the ſquare of the line drawn from another of the given points has the ſame ratio that a has to c, *to c, and ſo on, will be equal to the ſpace to which the ſum of the ſquares of the lines drawn from the two points found has the ſame ratio that twice a has to the ſum of a, b, c,* &c.

Let the number of the given points be three, and let *a*, *b*, *c* be equal to 1, 2, 3; and the theorem will be as follows.

Let there be three given points, two points may be found, ſuch, that if from the three given points and the two points found there be drawn right lines to any point, the ſquare of the line drawn from one of the given points together with twice the ſquare of the line drawn from another of the given points together with thrice the ſquare of the line drawn from the third given point, will be equal to the ſpace to which the ſum of the ſquares of the lines drawn from the two points found has the ſame ratio that two has to ſix, that is, will be equal to thrice the ſum of the ſquares of the lines drawn from the two points found; and ſo on.

PROPOSITION XIII.

THEOREM X.

Let there be any number of right lines given by poſition, and parallel to each other; a right line may be found parallel to the lines given by poſition, ſuch, that if from any point there be drawn a perpendicular to the right lines given by poſition and to the line found, the ſum of the ſquares of the lines intercepted between the point and the right lines given by poſition, will be equal to the multiple of the ſquare of the line intercepted between the point and the right line found, by the number of the right lines given by poſition, together with a given ſpace.

Let the number of the lines given by poſition be three, and the theorem will be as follows.

Let there be three lines given by poſition, and parallel to each other; a line may be found parallel to the right lines given by poſition, ſuch, that if from any point there be drawn a perpendicular to the right lines given by poſition

tion and to the line found, the ſum of the ſquares of the lines intercepted between the point and the three lines given by poſition, will be equal to thrice the ſquare of the line intercepted between the point and the line found, together with a given ſpace. And ſo on.

PROPOSITION XIV.

THEOREM XI.

Let there be any number of right lines interſecting each other in one point, and making all the angles round the point of interſection equal; and from any point let there be drawn perpendiculars to the right lines, and likewiſe let there be drawn a right line to the point of interſection: twice the ſum of the ſquares of the perpendiculars will be equal to the multiple of the ſquare of the line drawn to the point of interſection by the number of the lines.

Let the number of the lines be three, and the theorem will be as follows.

Let there be three right lines interſecting each other in a point, and making all the angles round the point of interſection equal; and from any point let there be drawn perpendiculars to the three lines, and likewiſe let there be drawn a right line to the point of interſection: twice the ſum of the ſquares of the perpendiculars will be equal to thrice the ſquare of the line drawn to the point of interſection.

Again, Let there be five right lines interſecting each other in one point, and making all the angles round the point of interſection equal; and from any point let there be drawn perpendiculars to the right lines, and likewiſe let there be drawn a right line to the point of interſection: twice the ſum of the ſquares of the perpendiculars will be equal to five times the ſquare of the line drawn to the point of interſection. And ſo on.

This theorem is very eaſily deduced from *Prop.* 4. by *Lem.* 2.

Cor. If there be any number of right lines interſecting each other in a given point, and making all the angles round the point of interſection equal; and from a point there be drawn perpendiculars to the right lines, and the ſum of the ſquares of the perpendiculars be equal to a given ſpace; the point from which the perpendiculars are drawn will be in the circumference of a given circle.

PROPOSITION XV.

THEOREM XII.

Let there be any number of right lines given by poſition interſecting each other in a point; two right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to all the right lines given by poſition, and likewiſe perpendiculars to the two right lines found, twice the ſum of the ſquares of the perpendiculars drawn to the right lines given by poſition, will be equal to the multiple of the ſum of the ſquares of the perpendiculars drawn to the two lines found by the number of the lines given by poſition.

Let the number of the lines given by poſition be three, and the theorem will be as follows.

Let there be three right lines given by poſition interſecting each other in a point; two right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the three lines given by poſition and to the two lines found, twice the ſum of the ſquares of the perpendiculars drawn to the three lines given by poſition, will be equal to thrice the ſum of the ſquares of the perpendiculars drawn to the two lines found. And ſo on.

PROPOSITION XVI.

THEOREM XIII.

Let there be any number of right lines given by poſition, that are neither all parallel, nor interſecting each other in one point; two right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe perpendiculars to the two right lines found, twice the ſum of the ſquares of the perpendiculars drawn to the right lines given by poſition, will be equal to the multiple of the ſum of the

the ſquares of the perpendiculars drawn to the two right lines found by the number of the right lines given by poſition, together with a given ſpace.

Let the number of the lines given by poſition be three, and the theorem will be as follows.

Let there be three right lines given by poſition, that are neither all parallel, nor interſecting each other in one point; two right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the three lines given by poſition and to the two lines found, twice the ſum of the ſquares of the perpendiculars drawn to the three lines given by poſition, will be equal to thrice the ſum of the ſquares of the perpendiculars drawn to the two lines found, together with a given ſpace. And ſo on.

Cor. If the right lines given by poſition be ſo ſituated, as to form an equilateral figure circumſcribed either about a circle or ſemicircle; or, if the number of the lines given by poſition be even, and each two and two of the lines interſect each other at right angles;

the two right lines that may be found will interſect each other at right angles.

From the three laſt theorems the following theorem may be eaſily derived.

PROPOSITION XVII.

THEOREM XIV.

Let there be any number greater than three of right lines given by poſition; three right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to all the right lines given by poſition, and likewiſe to the three lines found, thrice the ſum of the ſquares of the perpendiculars drawn to the right lines given by poſition, will be equal to the multiple of the ſum of the ſquares of the perpendiculars drawn to the three lines found by the number of the lines given by poſition.

Let the number of the lines given by poſition be four, and the theorem will be as follows.

Let there be four lines given by poſition; three lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the four lines given by poſition and to the three lines found, thrice the ſum of the ſquares of the perpendiculars drawn to the four lines given by poſition, will be equal to four times the ſum of the ſquares of the perpendiculars drawn to the three lines found.

The four laſt theorems may be made more general thus.

PROPOSITION XVIII.

THEOREM XV.

Let there be any number of right lines given by poſition, and parallel to each other; and let a, b, c, &c. *be given magnitudes as many in number as there are right lines given by poſition; a right line may be found parallel to the right lines given by poſition, ſuch, that if from any point there be drawn a perpendicular*

dicular to the right lines given by poſition, and likewiſe to the right line found, the ſquare of the ſegment intercepted between the point and one of the right lines given by poſition, together with the ſpace to which the ſquare of the ſegment intercepted between the point and another of the lines given by poſition has the ſame ratio that a has to b, together with the ſpace to which the ſquare of the ſegment intercepted between the point and another of the lines given by poſition has the ſame ratio that a has to c, and ſo on, will be equal to the ſpace to which the ſquare of the ſegment intercepted between the point and the line found has the ſame ratio that a has to the ſum of a, b, c, &c. *together with a given ſpace.*

Let the number of the lines given by poſition be three, and let *a*, *b*, *c* be equal to 1, 3, 5; and the theorem will be as follows.

Let there be three lines given by poſition, and parallel to each other; a right line may be found parallel to the right lines given by poſition, ſuch, that if from any point there be drawn a perpendicular to the three right lines given by poſition, and likewiſe to the right line found, the ſquare of the ſegment intercepted between the point and one of the right lines given by poſition, together with thrice the ſquare of the ſegment intercepted between the point and another of the lines given by poſition, together with five times the ſquare of the ſegment intercepted between the point and the third line given by poſition, will be equal to nine times the ſquare of the ſegment intercepted between the point and the line found, together with a given ſpace. And ſo on.

Cor. Let there be any number of right lines given by poſition, and parallel to each other; a right line may be found that will be given by poſition, ſuch, that if from any point there be drawn right lines in given angles to the right lines given by poſition, and likewiſe there be drawn a perpendicular to the right line found, the ſum of the ſquares of the lines drawn in given angles to the right lines given by poſition, will be equal to the ſpace to which the ſquare of the perpendicular drawn to the line found has a given ratio, together with a given ſpace.

PROPOSITION XIX.

THEOREM XVI.

Let there be any number of right lines given by poſition interſecting each other in a point, and let a, b, c, &c. be given magnitudes as many in number as there are right lines given by poſition; two right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the two right lines found, the ſquare of the perpendicular drawn to one of the lines given by poſition, together with the ſpace to which the ſquare of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that a has to b, together with the ſpace to which the ſquare of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that a has to c, and ſo on, will be equal to the ſpace to which the ſum of the ſquares of the perpendiculars drawn to the two lines found

found has the ſame ratio that twice a has to the ſum of a, b, c, &c.

Let the number of the lines given by poſition be three, and let *a*, *b*, *c* be equal to 1, 2, 4; and the theorem will be as follows.

Let there be three right lines given by poſition interſecting each other in a point; two right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the three lines given by poſition and to the two lines found, the ſquare of the perpendicular drawn to one of the right lines given by poſition, together with twice the ſquare of the perpendicular drawn to another of the lines given by poſition, together with four times the ſquare of the perpendicular drawn to the third line given by poſition, will be equal to the ſpace to which the ſum of the ſquares of the perpendiculars drawn to the two lines found has the ſame ratio that two has to ſeven. And ſo on.

Cor. Let there be any number of right lines given by poſition interſecting each other in a point; two right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn right lines in given angles to all the right lines given by poſition, and likewiſe there be drawn perpendiculars to the two lines found, the ſum of the ſquares of the lines drawn in given angles to the right lines given by poſition, will be equal to the ſpace to which the ſum of the ſquares of the perpendiculars drawn to the two lines found has a given ratio.

PROPOSITION XX.

THEOREM XVII.

Let there be any number of right lines given by poſition, that are neither all parallel, nor interſecting each other in one point, and let a, b, c, &c. *be given magnitudes as many in number as there are right lines given by poſition; two right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the two right lines found, the ſquare of the perpendicular drawn to one of the lines given by poſition*

poſition, together with the ſpace to which the ſquare of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that a has to b, together with the ſpace to which the ſquare of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that a has to c, and ſo on, will be equal to the ſpace to which the ſum of the ſquares of the perpendiculars drawn to the two lines found has the ſame ratio that twice a has to the ſum of a, b, c, &c. *together with a given ſpace.*

Let the number of the lines given by poſition be three, and let *a*, *b*, *c* be equal to 1, 2, 3; and the theorem will be as follows.

Let there be three right lines given by poſition, that are neither all parallel, nor interſecting each other in one point; two right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the three lines given by poſition and to the two lines found, the ſquare of the perpendicular drawn to one of the right lines given by poſition, together with twice the ſquare of the perpendicular drawn to another of the lines given by poſition, together with thrice the ſquare of the perpendicular drawn to the third line given by poſition, will be equal to thrice the ſum of the ſquares of the perpendiculars drawn to the two lines found, together with a given ſpace. And ſo on.

COR. Let there be any number of right lines given by poſition, that are neither all parallel, nor interſecting each other in one point; two right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn right lines in given angles to all the right lines given by poſition, and likewiſe there be drawn perpendiculars to the two lines found, the ſum of the ſquares of the lines drawn in given angles to the right lines given by poſition, will be equal to the ſpace to which the ſum of the ſquares of the perpendiculars drawn to the two lines found has a given ratio, together with a given ſpace.

PROPOSITION XXI.

THEOREM XVIII.

Let there be any number of right lines given by poſition, and let a, b, c, &c. *be given magnitudes as many in number as there are right lines given by poſition; three right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to all the right lines given by poſition, and likewiſe there be drawn perpendiculars to the three lines found, the ſquare of the perpendicular drawn to one of the lines given by poſition, together with the ſpace to which the ſquare of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that a has to b, together with the ſpace to which the ſquare of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that a has to c, and ſo on, will be equal to the ſpace to which the ſum of the ſquares of the perpendiculars drawn to the three lines found has the ſame ratio that thrice a has to the ſum of a, b, c,* &c.

Let the number of the lines given by poſition

tion be four, and let *a*, *b*, *c*, *d* be equal to 1, 2, 3, 4; and the theorem will be as follows.

Let there be four right lines given by poſition; three right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the four lines given by poſition, and likewiſe there be drawn perpendiculars to the three lines found, the ſquare of the perpendicular drawn to one of the lines given by poſition, together with twice the ſquare of the perpendicular drawn to another of the lines given by poſition, together with thrice the ſquare of the perpendicular drawn to another of the lines given by poſition, together with four times the ſquare of the perpendicular drawn to the fourth line given by poſition, will be equal to the ſpace to which the ſum of the ſquares of the perpendiculars drawn to the three lines given by poſition has the ſame ratio that three has to ten.

Cor. Let there be any number of right lines given by poſition; three right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn right lines in given angles to all the right lines given by poſition, and likewiſe there be drawn perpendiculars to the three lines found, the ſum of the ſquares of the lines drawn in given angles to the right lines given by poſition, will be equal to the ſpace to which the ſum of the ſquares of the perpendiculars drawn to the three lines found has a given ratio.

PROPOSITION XXII.

THEOREM XIX.

Let there be any regular figure of a greater number of ſides than three circumſcribed about a circle, and from any point in the circumference of the circle let there be drawn perpendiculars to the ſides of the figure; twice the ſum of the cubes of the perpendiculars, will be equal to five times the multiple of the cube of the ſemidiameter of the circle by the number of the ſides of the figure.

Suppoſe, for example, a ſquare to be circumſcribed about a circle, and from any point in the circumference of the circle let there be drawn perpendiculars to the ſides of the ſquare; the ſum of the cubes of the perpendiculars, will be equal to ten times the cube of the ſemidiameter of the circle.

Again, Suppoſe a pentagon to be circumſcribed about a circle, and from any point in the circumference of the circle let there be drawn perpendiculars to the ſides of the pentagon; twice the ſum of the cubes of the perpendiculars will be equal to twenty five times the cube of the ſemidiameter of the circle. And ſo on.

PROPOSITION XXIII.

THEOREM XX.

Let there be any regular figure circumſcribed about a circle of a greater number of ſides than three, and from any point within the figure let there be drawn perpendiculars to the ſides of the figure, and likewiſe let there be drawn a right line to the centre of the circle; twice the ſum of the cubes of the perpendiculars drawn to the ſides of the figure, will be equal to twice the multiple of the cube

cube of the ſemidiameter of the circle by the number of the ſides of the figure, together with thrice the multiple by the ſame number of the ſolid, whoſe baſe is the ſquare of the line drawn to the centre, and altitude the ſemidiameter of the circle.

Suppoſe, for example, a ſquare to be circumſcribed about a circle, and from any point within the ſquare let there be drawn perpendiculars to the ſides of the ſquare, and likewiſe let there be drawn a right line to the centre of the circle; the ſum of the cubes of the perpendiculars drawn to the ſides of the ſquare, will be equal to four times the cube of the ſemidiameter of the circle, together with ſix times the ſolid, whoſe baſe is the ſquare of the line drawn to the centre, and altitude the ſemidiameter of the circle.

Again, Let there be a pentagon circumſcribed about a circle, and from any point within the pentagon let there be drawn perpendiculars to the ſides of the pentagon, and likewiſe let there be drawn a right line to the centre of the circle; twice the ſum of the cubes of the perpendiculars drawn to the ſides of the pentagon,

tagon, will be equal to ten times the cube of the ſemidiameter of the circle, together with fifteen times the ſolid, whoſe baſe is the ſquare of the line drawn to the centre, and altitude the ſemidiameter of the circle. And ſo on.

Cor. I. Let there be any regular figure of a greater number of ſides than three circumſcribed about a circle, and from a point within the figure let there be drawn perpendiculars to the ſides of the figure, and let the ſum of the cubes of the perpendiculars be invariable; the point from which the perpendiculars are drawn, will be in the circumference of a given circle.

Cor. II. Let there be two regular figures circumſcribed about a circle, and let the number of the ſides of each figure be greater than three, and from any point within both figures let there be drawn perpendiculars to the ſides of both figures; the ſum of the cubes of the perpendiculars drawn to the ſides of one of the figures, will be to the ſum of the cubes of the perpendiculars drawn to the ſides of the other figure, as the number of the ſides of the one to the number of the ſides of the other.

Suppoſe, for example, a ſquare and a pentagon to be circumſcribed about a circle, and from any point within the ſquare and pentagon let there be drawn perpendiculars to the ſides of the ſquare and the pentagon; the ſum of the cubes of the perpendiculars drawn to the ſides of the ſquare, will be to the ſum of the cubes of the perpendiculars drawn to the ſides of the pentagon, as four to five.

PROPOSITION XXIV.

THEOREM XXI.

Let there be any figure given by poſition of a greater number of ſides than four; four right lines may be found that will be given by poſition, ſuch, that if from any point within the figure there be drawn perpendiculars to the ſides of the figure, and likewiſe there be drawn perpendiculars to the four lines found, four times the ſum of the cubes of the perpendiculars drawn to the ſides of the figure, will be equal to the multiple of the ſum of the cubes of the perpendiculars drawn to the four lines found by the number of the ſides of the figure.

For example, Let the number of the ſides of the figure be five; and the theorem will be as follows.

Let there be any five-ſided figure given by poſition; four right lines may be found, ſuch, that if from any point within the figure there be drawn perpendiculars to the ſides of the figure, and likewiſe there be drawn perpendiculars to the four lines found, four times the ſum of the cubes of the perpendiculars drawn to the ſides of the figure, will be equal to five times the ſum of the cubes of the perpendiculars drawn to the four lines found.

Again, Let the number of the ſides of the figure be ſix; and the theorem will be as follows.

Let there be a ſix-ſided figure given by poſition; four right lines may be found that will be given by poſition, ſuch, that if from any point within the figure there be drawn perpendiculars to the ſides of the figure, and likewiſe there be drawn perpendiculars to the four lines found, four times the ſum of the cubes of the perpendiculars drawn to the ſides of the figure, will be equal to ſix times the ſum of the cubes of the perpendiculars drawn to the four lines found. And ſo on.

PROPOSITION XXV.

THEOREM XXII.

Let there be any figure given by poſition of a greater number of ſides than three, and let a, b, c, &c. *be given magnitudes as many in number as there are ſides in the figure; four right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the ſides of the figure, and likewiſe there be drawn perpendiculars to the four lines found, the cube of the perpendicular drawn to one of the ſides of the figure, together with the ſolid to which the cube of the perpendicular drawn to another of the ſides of the figure has the ſame ratio that a has to b, together with the ſolid to which the cube of the perpendicular drawn to another of the ſides of the figure has the ſame ratio that a has to c, and ſo on, will be equal to the ſolid to which the ſum of the*

the cubes of the perpendiculars drawn to the four lines found has the ſame ratio that four times a has to the ſum of a, b, c, &c.

For example, Let the number of the ſides of the figure be four, and let a, b, c, d be equal to 1, 2, 3, 5; and the theorem will be as follows.

Let there be any quadrilateral figure given by poſition; four right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the ſides of the quadrilateral figure, and likewiſe there be drawn perpendiculars to the four lines found, the cube of the perpendicular drawn to one of the ſides of the figure, together with twice the cube of the perpendicular drawn to another of the ſides of the figure, together with thrice the cube of the perpendicular drawn to another of the ſides of the figure, and ſo on, will be equal to the ſolid to which the ſum of the cubes of the perpendiculars drawn to the four lines found has the ſame ratio that four has to eleven.

Again, Let the number of the ſides of the figure be five, and let *a*, *b*, *c*, *d*, *e* be equal to 1, 3, 5, 7, 9; and the theorem will be as follows.

Let there be any five-ſided figure given by poſition; four right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the ſides of the figure, and likewiſe there be drawn perpendiculars to the four lines found, the cube of the perpendicular drawn to one of the ſides of the figure, together with thrice the cube of the perpendicular drawn to another of the ſides of the figure, together with five times the cube of the perpendicular drawn to another of the ſides of the figure, and ſo on, will be equal to the ſolid to which the ſum of the cubes of the perpendiculars drawn to the four lines found has the ſame ratio that four has to twenty five. And ſo on.

Cor. Let there be any figure of a greater number of ſides than three; four lines may be found that will be given by poſition, ſuch, that if from any point within the figure there be drawn right lines in given angles to the ſides of the figure, and likewiſe there be drawn perpendiculars to the four lines found, the ſum of the cubes of the lines drawn in given angles to the ſides of the figure, will be to the ſum of the cubes of the perpendiculars drawn to the four lines found in a given ratio.

PROPOSITION XXVI.

THEOREM XXIII.

Let there be any regular figure inſcribed in a circle, and from all the angles of the figure let there be drawn right lines to any point in the circumference of the circle; the ſum of the fourth powers of the chords will be equal to 6 times the multiple of the fourth power of the ſemidiameter of the circle by the number of the ſides of the figure.

Suppoſe, for example, an equilateral triangle to be inſcribed in a circle, and from the angles of the triangle let there be drawn right lines to any point in the circumference of the circle; the ſum of the fourth powers of the chords will be equal to 18 times the fourth power of the ſemidiameter of the circle.

Again, Suppoſe a ſquare to be inſcribed in a circle, and from all the angles of the ſquare let there be drawn right lines to any point in the circumference of the circle; the ſum of the fourth powers of the chords, will be equal to 24 times the fourth power of the ſemidiameter of the circle. And ſo on.

PROPOSITION XXVII.

THEOREM XXIV.

Let there be any regular figure inſcribed in a circle, and from all the angles of the figure and the centre of the circle let there be drawn right lines to any point; the ſum of the fourth powers of the lines drawn from the angles of the figure, will be equal to the multiple by the number of the ſides of the figure of the fourth power of the ſemidiameter of the circle, together with 4 times the multiple by the ſame number of the fourth power of the line whoſe ſquare is equal to the rectangle contained by the ſemidiameter and the line drawn from the centre, together with the multiple by the ſame number of the fourth

fourth power of the line drawn from the centre.

Suppoſe, for example, an equilateral triangle to be inſcribed in a circle, and from all the angles of the triangle and the centre of the circle let there be drawn right lines to any point; the ſum of the fourth powers of the lines drawn from the angles of the triangle, will be equal to 3 times the fourth power of the ſemidiameter of the circle, together with 12 times the fourth power of the line whoſe ſquare is equal to the rectangle contained by the ſemidiameter and the line drawn from the centre, together with 3 times the fourth power of the line drawn from the centre.

Again, Suppoſe a ſquare to be inſcribed in a circle, and from all the angles of the ſquare and the centre of the circle let there be drawn right lines to any point; the ſum of the fourth powers of the lines drawn from the angles of the ſquare, will be equal to 4 times the fourth power of the ſemidiameter of the circle, together with 16 times the fourth power of the line whoſe ſquare is equal to the rectangle contained by the ſemidiameter and the line drawn from the centre, together with 4 times the fourth power of the line drawn from the centre.

Cor. I. Let there be two circles about the ſame centre, and from all the angles of any regular figure inſcribed in one of the circles let there be drawn right lines to any point in the circumference of the other; the ſum of the fourth powers of theſe lines will be invariable.

Cor. II. Let there be two regular figures inſcribed in a circle, and from all the angles of both figures let there be drawn right lines to any point; the ſum of the fourth powers of the lines drawn from all the angles of one of the figures, will be to the ſum of the fourth powers of the lines drawn from the angles of the other figure, as the number of the ſides of the one to the number of the ſides of the other.

Suppoſe, for example, an equilateral triangle and a ſquare to be inſcribed in a circle, and from all the angles of both figures let there be drawn right lines to any point; the ſum of the fourth powers of the lines drawn from the angles of the triangle, will be to the ſum of the fourth powers of the lines drawn from the angles of the ſquare, as three to four. And ſo on.

PROPOSITION XXVIII.

THEOREM XXV.

Let there be any regular figure of a greater number of ſides than four circumſcribed about a circle, and from any point in the circumference of the circle let there be drawn perpendiculars to the ſides of the figure; 8 *times the ſum of the fourth powers of the perpendiculars, will be equal to* 35 *times the multiple by the number of the ſides of the figure of the fourth power of the ſemidiameter of the circle.*

Suppoſe, for example, a pentagon to be circumſcribed about a circle, and from any point in the circumference of the circle let there be drawn perpendiculars to the ſides of the pentagon; 8 times the ſum of the fourth powers of the perpendiculars, will be equal to 175 times the fourth power of the ſemidiameter of the circle.

Again, Let there be a hexagon circumſcribed about a circle, and from any point in the circumference of the circle let there be drawn perpendiculars to the ſides of the hexagon; 4 times the ſum of the fourth powers of the perpendiculars, will be equal to 105 times the fourth power of the ſemidiameter of the circle. And ſo on.

PROPOSITION XXIX.

THEOREM XXVI.

Let there be any regular figure of a greater number of ſides than four circumſcribed about a circle, and from any point let there be drawn perpendiculars to the ſides of the figure, and likewiſe a right line to the centre of the circle; 8 *times the ſum of the fourth powers of the perpendiculars, will be equal to* 8 *times the multiple by the number of the ſides of the figure of the fourth power of the ſemidiameter of the circle, together with* 24 *times the multiple by the ſame number of the fourth power of the line whoſe ſquare is equal to* *to the rectangle contained by the ſemidiameter and the line drawn to the centre, together with 3 times the multiple of the fourth power of the line drawn to the centre of the circle by the number of the ſides of the figure.*

Suppoſe, for example, a pentagon to be circumſcribed about a circle, and from any point let there be drawn perpendiculars to the ſides of the pentagon, and likewiſe a line to the centre of the circle; 8 times the ſum of the fourth powers of the perpendiculars drawn to the ſides of the pentagon, will be equal to 40 times the fourth power of the ſemidiameter of the circle, together with 120 times the fourth power of the line whoſe ſquare is equal to the rectangle contained by the ſemidiameter and the line drawn to the centre of the circle, together with 15 times the fourth power of the line drawn to the centre. And ſo on.

Cor. I. Let there be two circles about the ſame centre, and about one of the circles let there be any regular figure of a greater number of ſides than four circumſcribed; if from any point in the circumference of the other there be drawn perpendiculars to the ſides of the figure, the ſum of the fourth powers of the perpendiculars will be invariable.

Cor. II. Let there be two regular figures circumſcribed about a circle, and let the number of the ſides of each figure be greater than four, and from any point let there be drawn perpendiculars to the ſides of both figures; the ſum of the fourth powers of the perpendiculars drawn to the ſides of one of the figures, will be to the ſum of the fourth powers of the perpendiculars drawn to the ſides of the other, as the number of the ſides of the one to the number of the ſides of the other.

Suppoſe, for example, a pentagon and hexagon to be circumſcribed about a circle, and from any point let there be drawn perpendiculars to the ſides of both figures; the ſum of the fourth powers of the perpendiculars drawn to the ſides of the pentagon, will be to the ſum of the fourth powers of the perpendiculars drawn to the ſides of the hexagon, as 5 to 6. And ſo on.

PROPOSITION XXX.

THEOREM XXVII.

Let there be any number of given points; two right lines may be found that will be given by poſition, and likewiſe a point may be found, ſuch, that if from all the given points and the point found there be drawn right lines to any point, and from the point to which the right lines are drawn there be drawn perpendiculars to the two right lines found, the ſum of the fourth powers of the lines drawn from the given points, will be equal to the multiple by the number of the given points of the fourth power of the line drawn from the point found, together with the fourth power of the line whoſe ſquare is a mean proportional between the ſum of the ſquares of the perpendiculars and a certain given ſpace, together with the fourth power of a certain given line.

Let the number of the given points be two; and the theorem will be as follows.

Let there be two given points; two right lines may be found that will be given by poſition, and likewiſe a point may be found, ſuch, that if from the two given points and the point found there be drawn right lines to any point, and from the point to which the lines are drawn there be drawn perpendiculars to the two right lines found, the ſum of the fourth powers of the lines drawn from the two given points, will be equal to twice the fourth power of the line drawn from the point found, together with the fourth power of the line whoſe ſquare is a mean proportional between the ſum of the ſquares of the perpendiculars and a certain given ſpace, together with the fourth power of a certain given line.

Let the number of the given points be three; and the theorem will be as follows.

Let there be three points given; two right lines may be found that will be given by poſition, and likewiſe a point may be found, ſuch, that if from all the three given points and the point found there be drawn right lines to any point, and from the point to which the right lines are drawn there be drawn perpendiculars to the two right lines found, the ſum of the fourth powers of the lines drawn from the three given points, will be equal to thrice the fourth power of the line drawn from the point found, together with the fourth power of the line whoſe ſquare is a mean proportional between the ſum of the ſquares of the perpendiculars and a certain given ſpace, together with the fourth power of a certain given line.

This theorem may be made more general thus.

PROPOSITION XXXI.

THEOREM XXVIII.

Let there be any number of given points, and let a, b, c, &c. *be given magnitudes as many in number as there are given points; two right lines may be found that will be given by poſition, and likewiſe a point may be found, ſuch, that if from all the given points and the*

the point found there be drawn right lines to any point, and from the point to which the lines are drawn there be drawn perpendiculars to the two right lines found, the fourth power of the line drawn from one of the given points, together with the power to which the fourth power of the line drawn from another of the given points has the ſame ratio that a has to b, together with the power to which the fourth power of the line drawn from another of the given points has the ſame ratio that a has to c, and ſo on, will be equal to the power to which the fourth power of the line drawn from the point found has the ſame ratio that a has to the ſum of a, b, c, &c. *together with the fourth power of the line whoſe ſquare is a mean proportional between the ſum of the ſquares of the perpendiculars and a certain given ſpace, together with the fourth power of a certain given line.*

For example, Let the number of the given points be two, and let *a*, *b* be equal to 1, 2; and the theorem will be as follows.

Let there be two given points; two right lines may be found that will be given by poſition,

tion, and likewiſe a point may be found, ſuch, that if from the two given points there be drawn right lines to any point, and from the point to which the right lines are drawn there be drawn perpendiculars to the two right lines found, the fourth power of the line drawn from one of the given points together with twice the fourth power of the line drawn from the other given point, will be equal to thrice the fourth power of the line drawn from the point found, together with the fourth power of the line whoſe ſquare is a mean proportional between the ſum of the ſquares of the perpendiculars drawn to the two lines found and a certain given ſpace, together with the fourth power of a certain given line.

Again, Let the number of the given points be three, and let a, b, c be equal to 1, 2, 3; and the theorem will be as follows.

Let there be three given points; two right lines may be found that will be given by poſition, and likewiſe a point may be found, ſuch, that if from all the given points and the point found there be drawn right lines to any point, and from the point to which the lines are drawn there be drawn perpendiculars to the two right lines found, the fourth power of the line drawn from one of the given points, together with twice the fourth power of the line drawn from another of the given points, together with thrice the fourth power of the line drawn from the laſt of the given points, will be equal to ſix times the fourth power of the line drawn from the point found, together with the fourth power of the line whoſe ſquare is a mean proportional between the ſum of the ſquares of the perpendiculars drawn to the two lines found and a certain given ſpace, together with the fourth power of a certain given line.

PROPOSITION XXXII.

THEOREM XXIX.

Let there be any number greater than three of given points; three points may be found, ſuch, that if from all the given points and the three points found there be drawn right lines to any point, thrice the ſum of the fourth powers of the lines drawn from the given points,

points, will be equal to the multiple by the number of the given points of the ſum of the fourth powers of the lines drawn from the three points found.

Let the number of the given points be four; and the theorem will be as follows.

Let there be four points given; three points may be found, ſuch, that if from the four given points and the three points found there be drawn right lines to any point, thrice the ſum of the fourth powers of the lines drawn from the four given points, will be equal to four times the ſum of the fourth powers of the lines drawn from the three points found.

Again, Let the number of the given points be five; and the theorem will be as follows.

Let there be five given points; three points may be found, ſuch, that if from the five given points and the three points found there be drawn right lines to any point, thrice the ſum of the fourth powers of the lines drawn from the five given points, will be equal to five times the ſum of the fourth powers of the lines drawn from the three points found. And ſo on.

PROPOSITION XXXIII.

THEOREM XXX.

Let there be any number greater than two of given points, and let a, b, c, d, &c. be given magnitudes as many in number as there are given points; three points may be found, ſuch, that if from all the given points and the three points found there be drawn right lines to any point, the fourth power of the line drawn from one of the given points, together with the power to which the fourth power of the line drawn from another of the given points has the ſame ratio that a has to b, together with the power to which the fourth power of the line drawn from another of the given points has the ſame ratio that a has to c, and ſo on, will be to the ſum of the fourth powers of the lines drawn from the three points found, as the ſum of a, b, c, &c. to thrice a.

For example, Let the number of the given points be three, and let *a*, *b*, *c* be equal to 1, 2, 4; and the theorem will be as follows.

Let there be three given points; three points may be found, ſuch, that if from the three given points and the three points found there be drawn right lines to any point, the fourth power of the line drawn from one of the given points, together with twice the fourth power of the line drawn from another of the given points, together with four times the fourth power of the line drawn from the third given point, will be to the ſum of the fourth powers of the lines drawn from the three points found, as ſeven to three.

Again, Let the number of the given points be four, and let *a*, *b*, *c*, *d* be equal to 1, 2, 4, 6; and the theorem will be as follows.

Let there be four given points; three points may be found, ſuch, that if from the four given points and the three points found there be drawn right lines to any point, the fourth power of the line drawn from one of the given points, together with 2 times the fourth power of the line drawn from another of the given points, together with 4 times the fourth power of the line drawn from another of the given points, together with 6 times the fourth power of the line drawn from the fourth given point, will be to the ſum of the fourth powers of the lines drawn from the three points found, as thirteen to three. And ſo on.

PROPOSITION XXXIV.

THEOREM XXXI.

Let there be any number greater than two of right lines interſecting each other in a point, and making all the angles round the point of interſection equal, and from any point let there be drawn perpendiculars to the right lines, and likewiſe let there be drawn a right line to the point of interſection; 8 *times the ſum of the fourth powers of the perpendiculars drawn to the right lines, will be equal to* 3 *times the multiple of the fourth power of the line drawn to the point of interſection by the number of the right lines.*

Let the number of the lines be three; and the theorem will be as follows.

Let there be three right lines interſecting each other in a point, and making all the angles

round

round the point of interſection equal, and from any point let there be drawn perpendiculars to the right lines, and likewiſe let there be drawn a right line to the point of interſection; 8 times the ſum of the fourth powers of the perpendiculars, will be equal to 9 times the fourth power of the line drawn to the point of interſection.

Again, Let the number of the lines be five; and the theorem will be as follows.

Let there be five right lines interſecting each other in a point, and making all the angles round the point of interſection equal, and from any point let there be drawn perpendiculars to the right lines, and likewiſe let there be drawn a right line to the point of interſection; 8 times the ſum of the fourth powers of the perpendiculars, will be equal to 15 times the fourth power of the line drawn to the point of interſection. And ſo on.

Cor. Let there be any number of right lines interſecting each other in a given point, and making all the angles round the point of interſection equal, and from a point let there be drawn perpendiculars to the right lines, and the ſum of the fourth powers of the perpendiculars be invariable; the point from which the perpendiculars are drawn, will be in the circumference of a given circle.

PROPOSITION XXXV.

THEOREM XXXII.

Let there be any number greater than three of right lines given by poſition, that are either all parallel to each other, or all interſecting each other in one point; three right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the three lines found, 3 times the ſum of the fourth powers of the perpendiculars drawn to the right lines given by poſition, will be equal to the multiple of the ſum of the fourth powers of the perpendiculars drawn to the three lines found by the number of the lines given by poſition.

Let the number of the lines be four; and the theorem will be as follows.

Let there be four right lines given by poſition, that are either all parallel to each other, or all interſecting each other in a point; three right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the four lines given by poſition, and likewiſe there be drawn perpendiculars to the three lines found, 3 times the ſum of the fourth powers of the perpendiculars drawn to the four lines given by poſition, will be equal to 4 times the ſum of the fourth powers of the perpendiculars drawn to the three lines found.

Again, Let the number of the lines be five; and the theorem will be as follows.

Let there be five right lines given by poſition, that are either all parallel to each other, or all interſecting each other in one point; three right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the five right lines given by poſition, and likewiſe there be drawn perpendiculars to the three lines found, 3 times the ſum of the fourth powers of the perpendiculars drawn to the five lines given by poſition, will be equal to 5 times the ſum of the fourth powers of the perpendiculars drawn to the three lines found. And ſo on.

PROPOSITION XXXVI.

THEOREM XXXIII.

Let there be any number greater than two of right lines given by poſition, that are either all parallel, or all interſecting each other in one point, and let a, b, c, d, &c. *be given magnitudes as many in number as there are right lines given by poſition; three right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the three lines found, the fourth power of the perpendicular drawn to one of the lines given by poſition, together with the power to which the fourth power of the perpendicular drawn to another of the lines given*

ven by poſition has the ſame ratio that a has to b, together with the power to which the fourth power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that a has to c, and ſo on, will be to the ſum of the fourth powers of the perpendiculars drawn to the three lines found, as the ſum of a, b, c, d, &c. *to thrice a.*

Let the number of the right lines given by poſition be three, and let *a*, *b*, *c* be equal to 1, 2, 5; and the theorem will be as follows.

Let there be three right lines given by poſition, that are either all parallel to each other, or all interſecting each other in one point; three right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the three right lines given by poſition, and likewiſe there be drawn perpendiculars to the three right lines found, the fourth power of the perpendicular drawn to one of the lines given by poſition, together with 2 times the fourth power of the perpendicular drawn to another of the lines given by poſition, together with 5 times the fourth power of the perpendicular drawn to the third line given by poſition, will be to the ſum of the fourth powers of the perpendiculars drawn to the three lines found, as eight to three.

Again, Let the number of the lines given by poſition be four, and let *a*, *b*, *c*, *d* be equal to 1, 2, 3, 4; and the theorem will be as follows.

Let there be four right lines given by poſition, that are either all parallel to each other, or all interſecting each other in one point; three right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the four right lines given by poſition, and likewiſe there be drawn perpendiculars to the three lines found, the fourth power of the perpendicular drawn to one of the lines given by poſition, together with 2 times the fourth power of the perpendicular drawn to another of the lines given by poſition, together with 3 times the fourth power of the perpendicular drawn to another of the lines given by poſition, together with 4 times the fourth power of the perpendicular drawn to the fourth line given by poſition, will be to the ſum of the fourth powers of the perpendiculars drawn to the three lines found, as ten to three. And ſo on.

Cor. Let there be any number greater than two of right lines given by poſition, that are either all parallel to each other, or all interſecting each other in one point; three right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn right lines in given angles to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the three lines found, the ſum of the fourth powers of the lines drawn in given angles to the right lines given by poſition, will be to the ſum of the fourth powers of the perpendiculars drawn to the three lines found in a given ratio.

PROPOSITION XXXVII.

Theorem XXXIV.

Let there be any number greater than five of right lines given by poſition, that are neither all parallel to each other, nor all interſecting each other in one point; five right lines may be found that will be given by poſition, ſuch,

ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the five lines found, five times the ſum of the fourth powers of the perpendiculars drawn to the right lines given by poſition, will be equal to the multiple of the ſum of the fourth powers of the perpendiculars drawn to the five lines found by the number of the right lines given by poſition.

Let the number of the lines given by poſition be ſix; and the theorem will be as follows.

Let there be ſix right lines given by poſition, that are neither all parallel to each other, nor all interſecting each other in one point; five right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the ſix lines given by poſition, and likewiſe there be drawn perpendiculars to the five lines found, five times the ſum of the fourth powers of the perpendiculars drawn to the ſix lines given by poſition, will be equal to ſix times the ſum of the fourth powers of the perpendiculars drawn to the five lines found.

Again, Let the number of the lines given by poſition be ſeven; and the theorem will be as follows.

Let there be ſeven right lines given by poſition, that are neither all parallel to each other, nor interſecting each other in one point; five right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the ſeven lines given by poſition, and likewiſe there be drawn perpendiculars to the five lines found, five times the ſum of the fourth powers of the perpendiculars drawn to the ſeven lines given by poſition, will be equal to ſeven times the ſum of the fourth powers of the perpendiculars drawn to the five lines found. And ſo on.

PROPOSITION XXXVIII.

THEOREM XXXV.

Let there be any number greater than four of right lines given by poſition, that are neither all parallel to each other, nor interſecting each

each other in one point, and let a, b, c, d, &c. *be given magnitudes as many in number as there are right lines given by poſition; five right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to all the right lines given by poſition, and likewiſe there be drawn perpendiculars to the five lines found, the fourth power of the perpendicular drawn to one of the lines given by poſition, together with the power to which the fourth power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that a has to b, together with the power to which the fourth power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that a has to c, and ſo on, will be to the ſum of the fourth powers of the perpendiculars drawn to the five lines found, as the ſum of a, b, c, d,* &c. *to five times a.*

Let the number of the lines given by poſition be five, and let *a*, *b*, *c*, *d*, *&c.* be equal to 1, 2, 3, 4, 6; and the theorem will be as follows.

Let there be five right lines given by poſition, that are neither all parallel to each other, nor interſecting each other in one point; five right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the five right lines given by poſition, and likewiſe there be drawn perpendiculars to the five lines found, the fourth power of the perpendicular drawn to one of the lines given by poſition, together with 2 times the fourth power of the perpendicular drawn to another of the lines given by poſition, together with 3 times the fourth power of the perpendicular drawn to another of the lines given by poſition, and ſo on, will be to the ſum of the fourth power of the perpendiculars drawn to the five lines found, as ſixteen to five.

Again, Let the number of the lines given by poſition be ſix, and let *a*, *b*, *c*, *d*, *&c.* be equal to 1, 3, 5, 7, 9, 11; and the theorem will be as follows.

Let there be ſix right lines given by poſition, that are neither all parallel to each other, nor all interſecting each other in one point; five right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the ſix lines given by poſition, and likewiſe there be drawn perpendiculars to the five lines found, the fourth power of the perpendicular drawn to one of the lines given by poſition, together with 3 times the fourth power of the perpendicular drawn to another of the lines given by poſition, together with 5 times the fourth power of the perpendicular drawn to another of the lines given by poſition, and ſo on, will be to the ſum of the fourth powers of the perpendiculars drawn to the five lines found, as thirty ſix to five. And ſo on.

Cor. Let there be any number greater than four of right lines given by poſition, that are neither all parallel to each other, nor all interſecting each other in one point; five right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn right lines in given angles to all the lines given by poſition, and likewiſe there be drawn perpendiculars to the five lines found, the ſum of the fourth powers of the lines drawn in given angles to the right lines given by poſition, will be to the ſum of the fourth powers of the perpendiculars drawn to the five lines found in a given ratio.

PROPOSITION XXXIX.

THEOREM XXXVI.

Let there be any regular figure circumſcribed about a circle; and let the number of the ſides of the figure be m, and let n be any number leſs than m; let r be the ſemidiameter of the circle; and from any point in the circumference of the circle let there be drawn perpendiculars to the ſides of the figure, the ſum of the n powers of the perpendiculars will be equal to $m \times \frac{1.3.5.7\ldots 2n-1}{1.2.3.4\ldots n} \times r^n$

The numbers in the numerator are to be continued till the laſt number be $2n-1$, and are to be continually multiplied into one another; the numbers in the denominator are to be continued till the laſt number be n, and are to be continually multiplied into one another.

For example, Let $m = 6$, and $n = 5$; and the theorem will be as follows.

Let there be a hexagon circumſcribed about a circle, and from any point in the circumference of the circle let there be drawn perpendiculars to the ſides of the hexagon, and let r be the ſemidiameter of the circle; the ſum of the fifth powers of the perpendiculars will be equal to $6 \times \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \times r^5 = \frac{189}{4} r^5$.

Again, Let $m = 8$, and $n = 6$; and the theorem will be as follows.

Let there be an octagon circumſcribed about a circle, and from any point in the circumference of the circle let there be drawn perpendiculars to the ſides of the octagon, and let r be the ſemidiameter of the circle; the ſum of the ſixth powers of the perpendiculars will be equal to $8 \times \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} \times r^6 = \frac{231}{2} r^6$. And ſo on.

PROPOSITION XL.

THEOREM XXXVII.

Let there be any regular figure circumſcribed about a circle, and let m be the number of *the ſides of the figure; let* n *be any number leſs than* m, *and let* r *be the ſemidiameter of the circle; and from any point (within the figure if* n *be an odd number, but if even, from any point either within or without) let there be drawn perpendiculars to the ſides of the figure; and likewiſe let there be drawn a right line to the centre of the circle, and let* v *be the line drawn to the centre; let* a *be the coefficient of the third term of a binomial raiſed to the* n *power,* b *the coefficient of the fifth term,* c *the coefficient of the ſeventh term, and ſo on; let* $A = a \times \frac{1}{2}$, $B = b \times \frac{1.\ 3}{2.\ 4}$, $C = c \times \frac{1.\ 3.\ 5}{2.\ 4.\ 6}$, *and ſo on: the ſum of the* n *powers of the perpendiculars will be equal to* $mr^n + mAv^2\,r^{n-2} + mBv^4r^{n-4} + mCv^6\,r^{n-6} +$ *&c.*

For example, Let $m = 6$, and $n = 5$; and becauſe the coefficients of the terms of a binomial raiſed to the fifth power are 1, 5, 10, 10, 5, 1, therefore $a = 10$, $b = 5$; and therefore $A = 5$, $B = \frac{15}{8}$. Therefore the theorem will be as follows.

Let there be a hexagon circumſcribed about a circle, and from a point within the hexagon let there be drawn perpendiculars to the ſides of the hexagon, and likewiſe a right line to the centre of the circle, and let r be the ſemidiameter of the circle, and v the line drawn to the centre; the ſum of the fifth powers of the perpendiculars drawn to the ſides of the hexagon will be equal to $6r^5 + 30r^3v^2 + \frac{45}{4}rv^4$

Again, Let $m = 8$, and $n = 6$; and becauſe the coefficients of the terms of a binomial raiſed to the ſixth power are 1, 6, 15, 20, 15, 6, 1, therefore $a = 15$, $b = 15$, $c = 1$; and therefore $A = \frac{15}{2}$, $B = \frac{45}{8}$, $C = \frac{5}{16}$. Therefore the theorem will be as follows.

Let there be an octagon circumſcribed about a circle, and from any point let there be drawn perpendiculars to the ſides of the octagon, and likewiſe a right line to the centre of the circle, and let r be the ſemidiameter of the circle, and v the line drawn to the centre; the ſum of the ſixth powers of the perpendiculars drawn to the ſides of the octagon, will be equal to $8r^6 + 60r^4v^2 + 45r^2v^4 + \frac{5}{2}v^6$. And ſo on.

Cor. I. Let there be any regular figure

circumſcribed about a circle, and let m be the number of the ſides of the figure, and n any number leſs than m; and from a point (within the figure if n be an odd number, but if n be even, from any point either within or without) let there be drawn perpendiculars to the ſides of the figure, and the ſum of the n powers of the perpendiculars be invariable; the point from which the perpendiculars are drawn, will be in the circumference of a given circle.

Cor. II. Let there be two regular figures circumſcribed about a circle, and let the number of the ſides of each figure be greater than n; and from any point (within both figures if n be an odd number, but if n be even, from any point either within or without) let there be drawn perpendiculars to the ſides of both figures; the ſum of the n powers of the perpendiculars drawn to the ſides of one of the figures, will be to the ſum of the n powers of the perpendiculars drawn to the ſides of the other figure, as the number of the ſides of the one to the number of the ſides of the other.

PROPOSITION XLI.

THEOREM XXXVIII.

Let there be any regular figure inſcribed in a circle, and let the number of the ſides of the figure be m; and let n be any number leſs than m; let r be the ſemidiameter of the circle; and from all the angles of the figure let there be drawn right lines to any point in the circumference of the circle: the ſum of the 2n powers of the chords, will be equal to $m \times \frac{1.3.5.7\ldots 2n-1}{1.2.3.4\ldots n} \times 2^n r^{2n}$.

The numbers in the numerator are to be continued till the laſt number be $2n-1$, and are to be continually multiplied into one another; the numbers in the denominator are to be continued till the laſt number be n, and are to be continually multiplied into one another.

For example, Let $m = 4$, and $n = 3$; and the theorem will be as follows.

Let there be a ſquare inſcribed in a circle, and from all the angles of the ſquare let there be drawn right lines to any point in the circumference of the circle; let r be the ſemidiameter of the circle: the ſum of the ſixth powers of the chords will be equal to $4 \times \frac{1 . 3 . 5}{1 . 2 . 3} \times 2^3 r^6 = 80 r^6$.

Again, Let $m = 5$, and $n = 4$; and the theorem will be as follows.

Let there be a pentagon inſcribed in a circle, and from all the angles of the pentagon let there be drawn right lines to any point in the circumference of the circle, and let r be the ſemidiameter of the circle; the ſum of the eighth powers of the chords will be equal to $5 \times \frac{1 . 3 . 5 . 7}{1 . 2 . 3 . 4} \times 2^4 r^8 = 350 r^8$. And ſo on.

PROPOSITION XLII.

THEOREM XXXIX.

Let there be any regular figure inſcribed in a circle, and let m be the number of the ſides of the figure, and n any number leſs than m; and from all the angles of the figure and the centre of the circle let there be drawn right lines

lines to any point; and let r be the ſemidiameter of the circle, and v the line drawn to the centre; let a be the coefficient of the ſecond term of a binomial raiſed to the n power, b the coefficient of the third term, c the coefficient of the fourth term, and ſo on: the ſum of the 2n powers of the lines drawn from the angles of the figure will be equal to $mr^{2n} + ma^2v^2r^{2n-2} + mb^2v^4r^{2n-4} + mc^2v^6r^{2n-6} +$ *&c.*

For example, Let $m = 4$, and $n = 3$; and becauſe the coefficients of the terms of a binomial raiſed to the third power are 1, 3, 3, 1, therefore $a = 3$, $b = 3$, $c = 1$; and therefore $a^2 = 9$, $b^2 = 9$, $c^2 = 1$. Therefore the theorem will be as follows.

Let there be a ſquare inſcribed in a circle, and from all the angles of the ſquare and the centre of the circle let there be drawn right lines to any point; let r be the ſemidiameter of the circle, and v the line drawn to the centre: the ſum of the ſixth powers of the lines drawn from the angles of the figure, will be equal to $4r^6 + 36v^2r^4 + 36v^4r^2 + 4v^6$.

Again, Let $m = 5$, and $n = 4$; and becauſe the coefficients of the terms of a binomial raiſed to the fourth power are 1, 4, 6, 4, 1, therefore $a = 4$, $b = 6$, $c = 4$, $d = 1$; and therefore $a^2 = 16$, $b^2 = 36$, $c^2 = 16$, $d^2 = 1$. Therefore the theorem will be as follows.

Let there be a pentagon inſcribed in a circle, and from all the angles of the pentagon and the centre of the circle let there be drawn right lines to any point; let r be the ſemidiameter of the circle, and v the line drawn to the centre: the ſum of the fourth powers of the lines drawn from the angles of the pentagon, will be equal to $5r^8 + 80v^2r^6 + 180v^4r^4 + 80v^6r^2 + 5v^8$. And ſo on.

COR. I. Let there be any regular figure inſcribed in a circle, and let m be the number of the ſides of the figure, and n any number leſs than m; and from all the angles of the figure let there be drawn right lines to a point, and the ſum of the $2n$ powers of the lines drawn from the angles of the figure be invariable; the point to which the lines are drawn, will be in the circumference of a given circle.

COR. II. Let there be two regular figures inſcribed in a circle, and let the number of the ſides of each figure be greater than n; and from all the angles of both figures let there be drawn right lines to any point: the ſum of the $2n$ powers of the lines drawn from the angles of one of the figures, will be to the ſum of the $2n$ powers of the lines drawn from the angles of the other, as the number of the ſides of the one to the number of the ſides of the other.

PROPOSITION XLIII.

THEOREM XL.

Let there be any number of given points, and let m be their number; let n be any number leſs than $m-1$: there may be found $n+1$ points, ſuch, that if from all the given points and the points found there be drawn right lines to any point, the ſum of the $2n$ powers of the lines drawn from the given points, will be to the ſum of the $2n$ powers of the lines drawn from the points found, as m to $n+1$.

For example, Let $n=3$; and the theorem will be as follows.

Let there be any number greater than 4 of given points; and let m be the number of the given points: 4 points may be found, ſuch, that if from all the given points and the 4 points found there be drawn right lines to any point, the ſum of the ſixth powers of the lines drawn from the given points, will be to the ſum of the ſixth powers of the lines drawn from the 4 points found, as m to 4.

Again, Let $n = 4$; and the theorem will be as follows.

Let there be any number greater than 5 of given points; and let m be their number: 5 points may be found, ſuch, that if from the given points and the 5 points found there be drawn right lines to any point, the ſum of the eighth powers of the lines drawn from the given points, will be to the ſum of the eighth powers of the lines drawn from the 5 points found, as m to 5. And ſo on.

PROPOSITION XLIV.

THEOREM XLI.

Let there be any number of given points, and let m *be their number; let* n *be any number leſs than* m, *and let* a, b, c, &c. *be given magnitudes as many in number as there are given points: there may be found* $n+1$ *points, ſuch, that if from all the given points and the points found there be drawn right lines to any point, the* $2n$ *power of the line drawn from one of the given points, together with the power to which the* $2n$ *power of the line drawn from another of the given points has the ſame ratio that* a *has to* b, *together with the power to which the* $2n$ *power of the line drawn from another of the given points has the ſame ratio that* a *has to* c, *and ſo on, will be to the ſum of the* $2n$ *powers of the lines drawn from the points found, as the ſum of* a, b, c, &c. *to* $\overline{n+1} \times a$.

For example, Let $n = 3$; and the theorem will be as follows.

Let there be any number greater than 3 of given points; and let *a*, *b*, *c*, *&c.* be given magnitudes as many in number as there are given points: 4 points may be found, ſuch, that if from all the given points and the 4 points found there be drawn right lines to any point, the ſixth power of the line drawn from one of the given points, together with the power to which the ſixth power of the line drawn from another of the given points has the ſame ratio that *a* has to *b*, together with the power to which the ſixth power of the line drawn from another of the given points has the ſame ratio that *a* has to *c*, and ſo on, will be to the ſum of the ſixth powers of the lines drawn from the four points found, as the ſum of *a*, *b*, *c*, *&c.* to 4 *a*.

Again, Let $n = 4$; and the theorem will be as follows.

Let there be any number greater than 4 of given points; and let *a*, *b*, *c*, *&c.* be given magnitudes as many in number as there are given points: 5 points may be found, ſuch, that if from the given points and the 5 points found there be drawn right lines to any point, the eighth power of the line drawn from one of the given points, together with the power to which the eighth power of the line drawn from another of the given points has the ſame ratio that *a* has to *b*, together with the power to which the eighth power of the line drawn from another of the given points has the ſame ratio that *a* has to *c*, and ſo on, will be to the ſum of the eighth powers of the lines drawn from the 5 points found, as the ſum of *a*, *b*, *c*, *&c.* to 5 *a*.

PROPOSITION XLV.

THEOREM XLII.

Let there be any number of right lines interſecting each other in a point, and making all the angles round the point of interſection equal; let m be the number of the lines, and let n be any number leſs than m; and from any point let there be drawn perpendiculars to the right lines, and likewiſe a right line to the point of interſection; let v be the line drawn to the point of interſection: the ſum of the 2n powers of the perpendiculars.

diculars drawn to the right lines will be equal to $m \times \frac{1.3.5.7\ldots 2n-1}{1.2.3.4\ldots n.2^n} \times v^{2n}$.

The numbers in the numerator are to be continued till the laſt number be equal to $2n-1$, and are to be continually multiplied into one another; the numbers in the denominator are to be continued till the laſt number be equal to n, and are to be continually multiplied into one another, and their product by 2^n.

For example, Let $n = 3$; and the theorem will be as follows.

Let there be any number greater than 3 of right lines interſecting each other in a point, and making all the angles round the point of interſection equal; and let m be the number of the lines; and from any point let there be drawn perpendiculars to the right lines, and likewiſe let there be drawn a line to the point of interſection, and let v be the line drawn to the point of interſection: the ſum of the ſixth powers of the perpendiculars drawn to the right lines, will be equal to $m \times \frac{1.3.5}{1.2.3.2^3} \times v^6 = m \times \frac{5}{16} v^6$.

Again, Let $n=4$; and the theorem will be as follows.

Let there be any number greater than 5 of right lines interſecting each other in a point, and making all the angles round the point of interſection equal; and let the number of the lines be m; and from any point let there be drawn perpendiculars to the right lines, and likewiſe let there be drawn a right line to the point of interſection, and let v be the line drawn to the point of interſection: the ſum of the eighth powers of the perpendiculars drawn to the right lines, will be equal to $m \times \frac{1.3.5.7}{1.2.3.4.2^4} \times nv^8 = m \times \frac{35}{128} v^8$. And ſo on.

Cor. If there be any number of lines interſecting each other in a given point, and making all the angles round the point of interſection equal; and m be the number of the lines, and n any number leſs than m; and from a point there be drawn perpendiculars to the right lines; and the ſum of the $2n$ powers of the perpendiculars be invariable: the point from which the perpendiculars are drawn, will be in the circumference of a given circle.

PROPOSITION XLVI.

THEOREM XLIII.

Let there be any number of right lines given by poſition, that are either all parallel to each other, or interſecting each other in one point; and let m be the number of the lines, and n any number leſs than $m-1$: there may be found $n+1$ right lines that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the lines found, the ſum of the $2n$ powers of the perpendiculars drawn to the right lines given by poſition, will be to the ſum of the $2n$ powers of the perpendiculars drawn to the lines found, as m to $n+1$.

For example, Let $n=3$; and the theorem will be as follows.

Let there be any number greater than 4 of right lines given by poſition, that are either all parallel to each other, or interſecting each other in one point; and let m be the number of the right lines given by poſition: there may be found 4 right lines that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the lines found, the ſum of the ſixth powers of the perpendiculars drawn to the right lines given by poſition, will be to the ſum of the ſixth powers of the perpendiculars drawn to the lines found, as m to 4.

Again, Let $n = 4$; and the theorem will be as follows.

Let there be any number greater than 5 of right lines given by poſition, that are either all parallel to each other, or interſecting each other in one point; and let m be the number of the right lines given by poſition: there may be found 5 right lines that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the 5 lines found, the ſum of the eighth powers of the perpendiculars drawn to the right lines given by poſition, will be to the ſum of the eighth powers of the perpendiculars drawn to the 5 lines found, as m to 5. And ſo on.

PROPOSITION XLVII.

THEOREM XLIV.

Let there be any number of right lines given by poſition, that are either all parallel to each other, or interſecting each other in one point; let m be the number of the right lines given by poſition, and let n be any number leſs than m; let a, b, c, &c. be given magnitudes as many in number as there are right lines given by poſition: there may be found $n+1$ right lines that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the lines found, the $2n$ power of the perpendicular drawn to one of the lines given by poſition, together with the power to which the $2n$ power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that a has to b, together with the power to which the $2n$ power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that a has to c, and ſo on, will be to the

the ſum of the $2n$ *powers of the perpendiculars drawn to the lines found, as the ſum of* a, b, c, &c. *to* $\overline{n+1}\times a$.

For example, Let $n=3$; and the theorem will be as follows.

Let there be any number greater than 3 of right lines given by poſition, that are either all parallel to each other, or interſecting each other in one point; and let a, b, c, *&c.* be given magnitudes as many in number as there are right lines given by poſition: 4 right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the 4 lines found, the ſixth power of the perpendicular drawn to one of the right lines given by poſition, together with the power to which the ſixth power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that a has to b, together with the power to which the ſixth power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that a has to c, and ſo on, will be to the ſum of the ſixth powers of the perpendiculars drawn to the 4 lines found, as the ſum of *a*, *b*, *c*, *&c.* to 4*a*.

Again, Let $n = 4$; and the theorem will be as follows.

Let there be any number greater than 4 of right lines given by poſition, that are either all parallel to each other, or interſecting each other in one point; and let *a*, *b*, *c*, *&c.* be given magnitudes as many in number as there are right lines given by poſition: 5 right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the 5 lines found, the eighth power of the perpendicular drawn to one of the lines given by poſition, together with the power to which the eighth power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that *a* has to *b*, together with the power to which the eighth power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that *a* has to *c*, and ſo on, will be to the ſum of the eighth powers of the perpendiculars drawn to the 5 lines found, as the ſum of *a*, *b*, *c*, *&c.* to 5*a*. And ſo on.

Cor. Let there be any number of right lines given by poſition, that are either all parallel to each other, or interſecting each other in one point; let *m* be the number of the right lines given by poſition, and let *n* be any number leſs than *m*: there may be found $n+1$ right lines that will be given by poſition, ſuch, that if from any point there be drawn right lines in given angles to all the right lines given by poſition, and likewiſe there be drawn perpendiculars to the right lines found, the ſum of the $2n$ powers of the right lines drawn in given angles to the right lines given by poſition, will be to the ſum of the $2n$ powers of the perpendiculars drawn to the lines found in a given ratio.

PROPOSITION XLVIII.

THEOREM XLV.

Let there be any number of right lines given by poſition, that are neither all parallel to each other,

other, nor interſecting each other in one point; and let the number of the right lines given by poſition be m, and let n be any even number leſs than $m-1$*: there may be found* $n+1$ *right lines that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to all the right lines given by poſition, and likewiſe there be drawn perpendiculars to the right lines found, the ſum of the n powers of the perpendiculars drawn to the right lines given by poſition, will be to the ſum of the n powers of the perpendiculars drawn to the lines found, as m to* $n+1$.

For example, Let $n=6$; and the theorem will be as follows.

Let there be any number greater than 7 of right lines given by poſition, that are neither all parallel to each other, nor interſecting each other in one point; and let m be the number of the right lines given by poſition: there may be found 7 right lines that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the 7 lines found, the ſum of the ſixth powers of the perpendiculars drawn to the right lines given by poſition, will be to the ſum of the ſixth powers of the perpendiculars drawn to the 7 lines found, as *m* to 7.

Again, Let $n = 8$; and the theorem will be as follows.

Let there be any number greater than 9 of right lines given by poſition, that are neither all parallel to each other, nor interſecting each other in one point; and let *m* be the number of the right lines given by poſition: there may be found 9 right lines that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the 9 lines found, the ſum of the eighth powers of the perpendiculars drawn to the right lines given by poſition, will be to the ſum of the eighth powers of the perpendiculars drawn to the 9 lines found, as *m* to 9. And ſo on.

PROPOSITION XLIX.

THEOREM XLVI.

Let there be any number of right lines given by poſition, that are neither all parallel to each other, nor interſecting each other in one point; let m be the number of the right lines given by poſition, and let a, b, c, &c. *be given magnitudes as many in number as there are right lines given by poſition; let n be any even number leſs than m: there may be found n* + 1 *right lines that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the right lines found, the n power of the perpendicular drawn to one of the lines given by poſition, together with the power to which the n power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that a has to b, together with the power to which the n power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that a has to c, and ſo on, will be to the*

the ſum of the n powers of the perpendiculars drawn to the lines found, as the ſum of a, b, c, &c. to $\overline{n+1} \times a$.

For example, Let $n = 6$; and the theorem will be as follows.

Let there be any number greater than 6 of right lines given by poſition, that are neither all parallel to each other, nor interſecting each other in one point; and let *a*, *b*, *c*, *&c.* be given magnitudes as many in number as there are right lines given by poſition: there may be found 7 right lines that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the 7 lines found, the ſixth power of the perpendicular drawn to one of the lines given by poſition, together with the power to which the ſixth power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that *a* has to *b*, together with the power to which the ſixth power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that *a* has to *c*, and ſo on, will be to the ſum of the ſixth powers of the perpendiculars drawn the lines found, as the ſum of *a*, *b*, *c*, *&c.* to $7a$.

Again, Let $n = 8$; and the theorem will be as follows.

Let there be any number greater than 8 of right lines given by poſition, that are neither all parallel to each other, nor interſecting each other in one point; and let *a*, *b*, *c*, *&c.* be given magnitudes as many in number as there are right lines given by poſition: there may be found 9 right lines that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the lines found, the eighth power of the perpendicular drawn to one of the lines given by poſition, together with the power to which the eighth power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that *a* has to *b*, together with the power to which the eighth power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that *a* has to *c*, and ſo on, will be to the ſum of the eighth powers of the perpendiculars drawn to the lines found, as the ſum of *a*, *b*, *c*, *&c.* to 9*a*.

Cor. Let there be any number of right lines given by poſition, that are neither all parallel to each other, nor interſecting each other in one point; let *m* be the number of the lines given by poſition, and let *n* be any even number leſs than *m*: there may be found $n + 1$ right lines that will be given by poſition, ſuch, that if from any point there be drawn right lines in given angles to all the right lines given by poſition, and likewiſe there be drawn perpendiculars to the right lines found, the ſum of the *n* powers of the lines drawn in given angles to the right lines given by poſition, will be to the ſum of the *n* powers of the perpendiculars drawn to the right lines found in a given ratio.

PROPOSITION L.

THEOREM XLVII.

Let there be any figure given by poſition; let m be the number of the ſides of the figure, and let n be any odd number leſs than $m - 1$:

there

there may be found $n+1$ *right lines that will be given by poſition, ſuch, that if from any point within the figure there be drawn perpendiculars to the ſides of the figure, and likewiſe there be drawn perpendiculars to the lines found, the ſum of the* n *powers of the perpendiculars drawn to the ſides of the figure, will be to the ſum of the* n *powers of the perpendiculars drawn to the lines found, as* m *to* $n+1$.

For example, Let $n=5$; and the theorem will be as follows.

Let there be any figure given by poſition of a greater number of ſides than 6; and let m be the number of the ſides of the figure: there may be found 6 right lines that will be given by poſition, ſuch, that if from any point within the figure there be drawn perpendiculars to the ſides of the figure, and likewiſe there be drawn perpendiculars to the 6 lines found, the ſum of the fifth powers of the perpendiculars drawn to the ſides of the figure, will be to the ſum of the fifth powers of the perpendiculars drawn to the 6 lines found, as m to 6.

Again, Let $n=7$; and the theorem will be as follows.

Let there be any figure given by poſition of a greater number of ſides than 8; and let m be the number of the ſides of the figure: there may be found 8 right lines that will be given by poſition, ſuch, that if from any point within the figure there be drawn perpendiculars to the ſides of the figure, and likewiſe there be drawn perpendiculars to the 8 lines found, the ſum of the ſeventh powers of the perpendiculars drawn to the ſides of the figure, will be to the ſum of the ſeventh powers of the perpendiculars drawn to the 8 lines found, as m to 8. And ſo on.

PROPOSITION LI.

THEOREM XLVIII.

Let there be any figure given by poſition; and let m *be the number of the ſides of the figure, and let* n *be any odd number leſs than* m*; and let* a*,* b*,* c*, &c. be given magnitudes as many in number as there are ſides in the figure: there may be found* $n+1$ *right lines* *that will be given by poſition, ſuch, that if from any point within the figure there be drawn perpendiculars to the ſides of the figure, and likewiſe there be drawn perpendiculars to the lines found, the n power of the perpendicular drawn to one of the ſides of the figure, together with the power to which the n power of the perpendicular drawn to another of the ſides of the figure has the ſame ratio that a has to b, together with the power to which the n power of the perpendicular drawn to another of the ſides of the figure has the ſame ratio that a has to c, and ſo on, will be to the ſum of the n powers of the perpendiculars drawn to the lines found, as the ſum of* a, b, c *&c. to* $\overline{n+1} \times a$.

For example, Let $n = 5$; and the theorem will be as follows.

Let there be any figure given by poſition of a greater number of ſides than 5; and let a, b, c, &c. be given magnitudes as many in number as there are ſides in the figure: there may be found 6 right lines that will be given by poſition, ſuch, that if from any point within the figure there be drawn perpendiculars to the ſides of the figure, and likewiſe there be drawn perpendiculars to the lines found, the fifth power of the perpendicular drawn to one of the ſides of the figure, together with the power to which the fifth power of the perpendicular drawn to another of the ſides of the figure has the ſame ratio that *a* has to *b*, together with the power to which the fifth power of the perpendicular drawn to another of the ſides of the figure has the ſame ratio that *a* has to *c*, and ſo on, will be to the ſum of the fifth powers of the perpendiculars drawn to the lines found, as the ſum of *a*, *b*, *c*, *&c.* to 6*a*.

Again, Let $n = 7$; and the theorem will be as follows.

Let there be any figure given by poſition of a greater number of ſides than 7; and let *a*, *b*, *c*, *&c.* be given magnitudes as many in number as there are ſides in the figure: 8 lines may be found that will be given by poſition, ſuch, that if from any point within the figure there be drawn perpendiculars to the ſides of the figure, and likewiſe there be drawn perpendiculars to the 8 lines found, the ſeventh power of the perpendicular drawn to one of the ſides of the figure, together with the power to which the ſeventh power of the perpendicular drawn to another of the ſides of the figure has the ſame ratio that *a* has to *b*, together with the power to which the ſeventh power of the perpendicular drawn to another of the ſides of the figure has the ſame ratio that *a* has to *c*, and ſo on, will be to the ſum of the ſeventh powers of the perpendiculars drawn to the lines found, as the ſum of *a*, *b*, *c*, *&c.* to $8a$.

Cor. Let there be any figure given by poſition; and let *m* be the number of the ſides of the figure, and let *n* be any odd number leſs than *m*: there may be found $n+1$ right lines that will be given by poſition, ſuch, that if from any point within the figure there be drawn right lines in given angles to the ſides of the figure, and likewiſe there be drawn perpendiculars to the lines found, the ſum of the *n* powers of the lines drawn in given angles to the ſides of the figure, will be to the ſum of the *n* powers of the perpendiculars drawn to the lines found in a given ratio.

N. B. In the following theorems by taking a point always on the ſame ſide the right lines given by poſition, we are to underſtand that the point muſt not be taken on different ſides any one of the right lines given by poſition.

The two laſt theorems may be made more general thus.

PROPOSITION LII.

THEOREM XLIX.

Let there be any number of right lines given by poſition; and let m *be the number of the lines, and* n *any odd number leſs than* $m-1$: *there may be found* $n+1$ *right lines that will be given by poſition, ſuch, that if from any point always taken on the ſame ſide the right lines given by poſition there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the lines found, the ſum of the* n *powers of the perpendiculars drawn to the right lines given by poſition, will be to the ſum of the* n *powers of the perpendiculars drawn to the lines found, as* m *to* $n+1$.

For example, Let $n = 3$; and the theorem will be as follows.

Let there be any number greater than 4 of right lines given by poſition; and let m be the number of the lines: 4 right lines may be found that will be given by poſition, ſuch, that if from any point always taken on the ſame ſide the right lines given by poſition there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the 4 lines found, the ſum of the cubes of the perpendiculars drawn to the right lines given by poſition, will be to the ſum of the cubes of the perpendiculars drawn to the 4 lines found, as m to 4.

Again, Let $n = 5$; and the theorem will be as follows.

Let there be any number greater than 6 of right lines given by poſition; and let m be the number of the lines: 6 right lines may be found that will be given by poſition, ſuch, that if from any point always taken on the ſame ſide the right lines given by poſition there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the 6 lines found, the ſum of the ſixth powers of the perpendiculars drawn to the right lines given by poſition, will be to the ſum of the ſixth powers of the perpendiculars drawn to the 6 lines found, as *m* to 6. And ſo on.

PROPOSITION LIII.

THEOREM L.

Let there be any number of right lines given by poſition; and let m *be the number of the lines, and* n *any odd number leſs than* m; *and let* a, b, c, &c. *be given magnitudes as many in number as there are right lines given by poſition: there may be found* $n+1$ *right lines that will be given by poſition, ſuch, that if from any point always taken on the ſame ſide the right lines given by poſition there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the lines found, the* n *power of the perpendicular drawn to one of the lines given by poſition, together with the power to which the* n *power of the perpendicular drawn to another of the lines given*

ven by poſition has the ſame ratio that a *has to* b, *together with the power to which the* n *power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that* a *has to* c, *and ſo on, will be to the ſum of the* n *powers of the perpendiculars drawn to the lines found, as the ſum of* a, b, c, &c. *to* $\overline{n+1} \times a$.

For example, Let $n = 3$; and the theorem will be as follows.

Let there be any number greater than 4 of right lines given by poſition; and let a, b, c, *&c.* be given magnitudes as many in number as there are right lines given by poſition: 4 lines may be found that will be given by poſition, ſuch, that if from any point always taken on the ſame ſide the right lines given by poſition there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the lines found, the cube of the perpendicular drawn to one of the lines given by poſition, together with the ſolid to which the cube of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that a has to b, to *b*, together with the ſolid to which the cube of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that *a* has to *c*, and ſo on, will be to the ſum of the cubes of the perpendiculars drawn to the lines found, as the ſum of *a*, *b*, *c*, *&c.* to 4*a*.

Again, Let $n = 5$; and the theorem will be as follows.

Let there be any number greater than 6 of right lines given by poſition; and let *a*, *b*, *c*, *&c.* be given magnitudes as many in number as there are right lines given by poſition: 6 lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the lines found, the fifth power of the perpendicular drawn to one of the lines given by poſition, together with the power to which the fifth power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that *a* has to *b*, together with the power to which the fifth power of the perpendicular drawn to another of the lines given by poſition has the ſame ratio that *a* has *a* has to *c*, and ſo on, will be to the ſum of the fifth powers of the perpendiculars drawn to the lines found, as the ſum of *a*, *b*, *c*, *&c.* to 6*a*. And ſo on.

Cor. Let there be any number of right lines given by poſition; and let *m* be the number of the lines, and *n* any odd number leſs than *m*: there may be found $n + 1$ right lines that will be given by poſition, ſuch, that if from any point always taken on the ſame ſide the right lines given by poſition there be drawn lines in given angles to the right lines given by poſition, and likewiſe there be drawn perpendiculars to the lines found, the ſum of the *n* powers of the lines drawn in given angles to the right lines given by poſition, will be to the ſum of the *n* powers of the perpendiculars drawn to the lines found in a given ratio.

PROPOSITION LIV.

Let there be any number of right lines given by poſition, and parallel to each other; and let n be any given number; and from a point let there be drawn right lines in given angles *to all the right lines given by poſition; and let the ſum of the n powers of the lines drawn in given angles be invariable: the point from which the lines are drawn, will be in a right line given by poſition.*

For example, Let $n = 2$; and the propoſition will be as follows.

Let there be any number of right lines given by poſition, and parallel to each other; and from a point let there be drawn right lines in given angles to the right lines given by poſition; and let the ſum of the ſquares of the lines drawn in given angles be invariable: the point from which the lines are drawn, will be in a right line given by poſition.

Again, Let $n = 3$; and the propoſition will be as follows.

Let there be any number of right lines given by poſition, and parallel to each other; and from a point let there be drawn right lines in given angles to the right lines given by poſition; and let the ſum of the cubes of the lines drawn in given angles be invariable: the point from which the lines are drawn, will be in a right line given by poſition.

PROPOSITION LV.

Let there be any number of right lines given by poſition, that are not all parallel to each other; let $n+1$ be the number of the lines; and from a point let there be drawn perpendiculars to the right lines given by poſition; and let the ſum of the n powers of the perpendiculars be invariable: the point from which the perpendiculars are drawn, will be in an oval figure that is a line of the n order, or a line of an inferior order.

This is a locus not only to all the various oval figures of lines of the *n* order, but likewiſe to all the various oval figures of lines of any lower order; that is, the various caſes of this locus will comprehend all the various oval figures of lines of the *n* order, and likewiſe all the various oval figures of lines of any lower order.

For example, Let $n=3$; and the propoſition will be as follows.

Let there be four right lines given by poſition, that are not all parallel to each other;

and from a point let there be drawn perpendiculars to the right lines given by poſition; and let the ſum of the cubes of the perpendiculars be invariable: the point from which the perpendiculars are drawn, will be in an oval figure that is a line of the third order, or will be in an ellipſe or circle; and the various caſes of this locus will comprehend all the oval figures that are lines of the third order, and likewiſe the ellipſe and circle.

If the right lines given by poſition be the ſides of a ſquare, and the point from which the perpendiculars are drawn be within the ſquare, the point will be in a circle. But

If the right lines given by poſition be the ſides of a parallelogram, and the point from which the perpendiculars are drawn be taken within the parallelogram, the point will be in an ellipſe.

Again, Let $n = 4$; and the propoſition will be as follows.

Let there be five right lines given by poſition, that are not all parallel to each other; and from a point let there be drawn perpendiculars to the right lines given by poſition; and let the ſum of the fourth powers of the perpendiculars be invariable: the point from which the perpendiculars are drawn, will be in an oval figure that is a line of the fourth order, or will be in an oval figure that is a line of an inferior order.

This is a locus not only to all the various oval figures of lines of the fourth order, but likewiſe to all the oval figures of lines of the third and ſecond order; that is, the various caſes of this locus will comprehend all the various oval figures of lines of the fourth order, and likewiſe all the various oval figures of lines of the third order, and alſo the ellipſe and circle. And ſo on.

PROPOSITION LVI.

Let there be any number p of right lines given by poſition, and likewiſe let there be any number q of right lines given by poſition; and let all the lines be either parallel to each other, or all interſecting each other in one point; and let n be any number; and from a point let there be drawn right lines in given angles to all the right lines given by poſition;

ſition; and let the ſum of the n powers of the lines drawn in given angles to the firſt number of right lines given by poſition be to the ſum of the n powers of the lines drawn in given angles to the ſecond number of right lines given by poſition in a given ratio: the point from which the lines are drawn, will be in a right line given by poſition.

For example, Let $n = 2$; and the propoſition will be as follows.

Let there be p number of right lines given by poſition, and likewiſe let there be q number of right lines given by poſition; and let all the right lines be either parallel to each other, or all interſecting each other in one point; and from a point let there be drawn right lines in given angles to all the right lines given by poſition; and let the ſum of the ſquares of the lines drawn in given angles to the firſt number of right lines given by poſition be to the ſum of the ſquares of the lines drawn in given angles to the ſecond number of right lines given by poſition in a given ratio: the point from which the lines are drawn, will be in a right line given by poſition.

Again,

Again, Let $n = 3$; and the propoſition will be as follows.

Let there be any number p of right lines given by poſition, and likewiſe let there be any number q of right lines given by poſition; and let all the right lines be either parallel to each other, or all interſecting each other in one point; and from a point let there be drawn right lines in given angles to all the right lines given by poſition; and let the ſum of the cubes of the lines drawn in given angles to the firſt number of right lines given by poſition be to the ſum of the cubes of the right lines drawn in given angles to the ſecond number of right lines given by poſition in a given ratio: the point from which the lines are drawn in given angles, will be in a right line given by poſition.

PROPOSITION LVII.

Let there be any even number of right lines given by poſition; let $2n+2$ be the number of the lines; and from a point let there be drawn perpendiculars to the right lines given

ven by poſition; and let the ſum of the n powers of the perpendiculars drawn to half the number of right lines given by poſition be to the ſum of the n powers of the perpendiculars drawn to the remaining number of right lines given by poſition in a given ration: the point from which the perpendiculars are drawn, will be in a line of the n order, or will be in a line of an inferior order.

This is a locus not only to all the various lines of the *n* order, but likewiſe to all the various lines of any lower order; that is, the various caſes of this locus will comprehend all the various lines of the *n* order, and likewiſe all the various lines of any inferior order.

For example, Let $n = 3$; and the propoſition will be as follows.

Let there be 8 right lines given by poſition; and from a point let there be drawn perpendiculars to the right lines given by poſition; and let the ſum of the cubes of the perpendiculars drawn to four of the lines given by poſition be to the ſum of the cubes of the perpendiculars drawn to the other 4 lines given by poſition in a given ratio: the point from which the perpendiculars are drawn, will be in a line of the third order, or will be in a line of the ſecond order, or in a right line.

This is a locus not only to all the various lines of the third order, but likewiſe to all the lines of the ſecond order, and likewiſe to a right line; that is, the various caſes of this locus will comprehend all the lines of the third order, and all the lines of the ſecond order, and a right line.

Let there be two parallelograms given by poſition; and from a point within both figures let there be drawn perpendiculars to the ſides of the figures; and let the ſum of the cubes of the perpendiculars drawn to the ſides of the one be to the ſum of the cubes of the perpendiculars drawn to the ſides of the other in a given ratio: the point from which the perpendiculars are drawn, will be in a conic ſection; and if the figures be both ſquares, the point will be in a circle, or in a right line.

In many other caſes of this propoſition, the point will be in a conic ſection, and alſo in a circle, or in a right line.

Again, Let $n = 4$; and the propoſition will be as follows.

Let there be ten right lines given by poſition; and from a point let there be drawn perpendiculars to the right lines given by poſition; and let the ſum of the fourth powers of the perpendiculars drawn to five of the right lines given by poſition be to the ſum of the fourth powers of the perpendiculars drawn to the other five lines given by poſition in a given ratio: the point from which the perpendiculars are drawn, will be in a line of the fourth order, or will be in a line of the third or ſecond order, or will be in a right line.

This is a locus not only to all the various lines of the fourth order, but likewiſe to all the various lines of the third and ſecond order; that is, the various caſes of this locus will comprehend all the lines of the fourth order, and likewiſe all the lines of the third and ſecond order, and alſo a right line.

PROPOSITION LVIII.

Let there be any number of right lines given by poſition, that are not all parallel to each other; and let n be any number; and from a point let there be drawn right lines in given angles

angles to the right lines given by poſition; and let the ſum of the n powers of the lines drawn in given angles be invariable: the point from which the lines are drawn, will be in an oval figure that is a line of the n order, or a line of an inferior order.

If the number of the right lines given by poſition be greater than *n*, there may be found [by *Cor.* to 49. & 53.] $n+1$ right lines that will be given by poſition, ſuch, that if from any point there be drawn right lines in given angles to all the right lines given by poſition, and likewiſe there be drawn perpendiculars to the lines found, the ſum of the *n* powers of the lines drawn in given angles to the right lines given by poſition, will be to the ſum of the *n* powers of the perpendiculars drawn to the lines found, as the number of the lines given by poſition to $n+1$; and becauſe the ſum of the *n* powers of the lines drawn in given angles to the right lines given by poſition is invariable, therefore the ſum of the *n* powers of the perpendiculars drawn to the $n+1$ lines found will be invariable: Therefore [54.] the point from which the perpendiculars are drawn, will be in an oval figure that is a line of the *n* order, or a line of an inferior order: And therefore the point from which the lines are drawn in given angles to the right lines given by poſition, will be in an oval figure that is a line of the *n* order, or a line of an inferior order.

Let $n = 2$; and the propoſition will be as follows.

Let there be any number of right lines given by poſition that are not all parallel to each other; and from a point let there be drawn right lines in given angles to all the right lines given by poſition; and let the ſum of the ſquares of the lines drawn in given angles be invariable: the point from which the lines are drawn will be in an ellipſe.

Again, Let $n = 3$; and the propoſition will be as follows.

Let there be any number of right lines given by poſition that are not all parallel to each other; and from a point let there be drawn right lines in given angles to all the right lines given by poſition; and let the ſum of the cubes of the lines drawn in given angles be invariable: the point from which the lines are drawn in given angles, will be in an oval figure that is a line of the third order, or will be in an ellipſe or circle.

PROPOSITION LIX.

Let there be any number p of right lines given by poſition, and likewiſe let there be any number q of right lines given by poſition, and let n be any given number; and from a point let there be drawn right lines in given angles to all the right lines given by poſition; and let the ſum of the n powers of the lines drawn in given angles to the firſt number of right lines given by poſition be to the ſum of the n powers of the lines drawn in given angles to the ſecond number of right lines given by poſition in a given ratio: the point from which the lines are drawn will be in a line of the n order, or will be in a line of an inferior order.

Let $n = 2$; and the propoſition will be as follows.

Let there be any number p of right lines given by poſition, and likewiſe let there be any number q of right lines given by poſition; and from a point let there be drawn right lines in given angles to all the right lines given by poſition; and let the ſum of the ſquares of the lines drawn in given angles to the firſt number of right lines given by poſition be to the ſum of the ſquares of the lines drawn in given angles to the ſecond number of right lines given by poſition in a given ratio: the point from which the lines are drawn, will be in a line of the ſecond order, or in a right line; that is, the point will be in a conic ſection.

Let there be two circles given by poſition, and about each of the circles let there be a regular figure circumſcribed; and from a point let there be drawn perpendiculars to the ſides of both figures; and let the ſum of the ſquares of the perpendiculars drawn to the ſides of one of the figures be to the ſum of the ſquares of the perpendiculars drawn to the ſides of the other figure in a given ratio: the point from which the perpendiculars are drawn, will be in a circle, or in a right line. If the given ratio be the ſame with that of the number of the ſides of the firſt figure to the number of the ſides of the ſecond, the point from which the perpendiculars are drawn, will be in a right line given by poſition.

In many other caſes of this propoſition, the point will be either in a circle, or in a right line.

Again, Let $n = 3$; and the propoſition will be as follows.

Let there be any number p of right lines given by poſition, and likewiſe let there be any number q of right lines given by poſition; and from a point let there be drawn right lines in given angles to all the right lines given by poſition; and let the ſum of the cubes of the lines drawn in given angles to the firſt number of right lines given by poſition be to the ſum of the cubes of the lines drawn in given angles to the ſecond number of right lines given by poſition in a given ratio: the point from which the lines are drawn, will be in a line of the third order, or in a line of the ſecond order, or in a right line.

For example, Let there be a parallelogram given by poſition, and likewiſe let there be a regular figure of a greater number of ſides than three circumſcribed about a circle given by poſition; and from a point within both figures let there be drawn perpendiculars to the ſides of both figures; and let the ſum of the cubes of the perpendiculars drawn to the ſides of one of the figures be to the ſum of the cubes of the perpendiculars drawn to the ſides of the other figure in a given ratio: the point from which the perpendiculars are drawn, will be in a line of the ſecond order, or will be in a right line; that is, the point will be in a conic ſection.

Let there be two circles given by poſition, and about each of the circles let there be a regular figure of a greater number of ſides than three circumſcribed; and from a point within both figures let there be drawn perpendiculars to the ſides of both figures; and let the ſum of the cubes of the perpendiculars drawn to the ſides of one of the figures be to the ſum of the cubes of the perpendiculars drawn to the ſides of the other in a given ratio: the point from which the perpendiculars are drawn, will be in a circle, or in a right line.

If the multiple of the diameter of the firſt circle by the number of the ſides of the figure circumſcribed about it be to the multiple of the diameter of the ſecond circle by the number

ber of the ſides of the figure circumſcribed about it in the given ratio, that is, as the ſum of the cubes of the perpendiculars drawn to the ſides of the figure circumſcribed about the firſt circle to the ſum of the cubes of the perpendiculars drawn to the ſides of the figure circumſcribed about the ſecond circle; the point from which the perpendiculars are drawn, will be in a right line.

In many other caſes of this propoſition, the point from which the lines are drawn, will be in a circle, or in a right line.

The following propoſitions are a few properties of the circle that occurred when conſidering ſome of the foregoing propoſitions.

PROPOSITION LX. *Fig.* 19.

Let there be a circle given by poſition, and let A, B *be two given points; a point* C *may be found within the circle, ſuch, that if through the point* C *there be drawn any line meeting the circle in* D, E, *and* AD, BD, AE, BE *be joined, the rectangle* ADB *will be to the rectangle* AEB *as* CD *to* CE.

PRO-

PROPOSITION LXI. *Fig.* 20.

Let there be a circle given by poſition, and two points A, B *given; two right lines* DE, DF *may be found, ſuch, that if from the points* A, B *there be drawn* AG, BG *to any point* G *in the circumference of the circle, and from the point* G *there be drawn* GH, GK *perpendicular to* DE, DF, *the ſum of the ſquares of* GH, GK *will be to the rectangle* AGB *as the rectangle* AGB *to a certain given ſpace; that is, the rectangle* AGB *will be a mean proportional between the ſum of the ſquares of* GH, GK *and a certain given ſpace.*

PROPOSITION LXII. *Fig.* 21.

Let there be a circle given by poſition, and let there be two right lines AB, AC *given by poſition; and let a, b be two given magnitudes: a point* D *may be found, ſuch, that if through the point* D *there be drawn any right line meeting the circle in* E, F, *and from the point* E *there be drawn* EG, EH *perpendicular to* AB, AC, *and from the point* F *there be drawn likewiſe* FK, FL *perpendicular to* AB, AC, *the ſquare of* EG *together with the ſpace to which the ſquare of* EH *has the ſame ratio that a has to b, will be to the ſquare of* FK *together with the ſpace to which the ſquare of* FL *has the ſame ratio that a has to b, as the ſquare of* ED *to the ſquare of* DF.

PROPOSITION LXIII. *Fig.* 22.23.

Let there be a circle given by poſition, and let AB, AC *be two right lines given by poſition, and let the angle* BAC *be equal to two angles of an equilateral triangle; two right lines* DE, DF *may be found that will be given by poſition, ſuch, that if from any point* G *in the circumference of the circle within the angle* BAC *there be drawn* GH, GK *perpendicular to* AB, AC, *and likewiſe there be drawn* GL, GM *perpendicular to* DE, DF, *the ſum of the cubes of* GH, GK *will be equal to a ſolid whoſe baſe is the ſum of the ſquares of* GL, GM, *and altitude a given line.*

PRO-

PROPOSITION LXIV. *Fig.* 24.

Let there be a circle given by poſition, and AB, AC *two right lines given by poſition, and let the angle* BAC *be equal to two angles of an equilateral triangle, and let the circle be contained within the angle* BAC; *a point* D *may be found, ſuch, that if through the point* D *there be drawn any line meeting the circle in* E, F, *and from the point* E *there be drawn* EG, EH *perpendicular to* AB, AC, *and likewiſe there be drawn* FK, FL *perpendicular to* AB, AC, *the ſum of the cubes of* EG, EH, *will be to the ſum of the cubes of* FK, FL, *as the ſquare of* DE *to the ſquare of* DF.

There are many properties of the circle and conic ſections ſimilar to theſe, that will naturally occur to ſuch as conſider ſome of the foregoing propoſitions, and that may be of conſiderable uſe in ſolving ſeveral problems, that at firſt view would ſeem to require a line of a high order, when the ſolution may be eaſily had by a conic ſection.

For example, *Fig.* 20. Let there be a cir-

perpendicular to AB, AC, *and from the point* F *there be drawn likewiſe* FK, FL *perpendicular to* AB, AC, *the ſquare of* EG *together with the ſpace to which the ſquare of* EH *has the ſame ratio that a has to b, will be to the ſquare of* FK *together with the ſpace to which the ſquare of* FL *has the ſame ratio that a has to b, as the ſquare of* ED *to the ſquare of* DF.

PROPOSITION LXIII. *Fig.* 22.23.

Let there be a circle given by poſition, and let AB, AC *be two right lines given by poſition, and let the angle* BAC *be equal to two angles of an equilateral triangle; two right lines* DE, DF *may be found that will be given by poſition, ſuch, that if from any point* G *in the circumference of the circle within the angle* BAC *there be drawn* GH, GK *perpendicular to* AB, AC, *and likewiſe there be drawn* GL, GM *perpendicular to* DE, DF, *the ſum of the cubes of* GH, GK *will be equal to a ſolid whoſe baſe is the ſum of the ſquares of* GL, GM, *and altitude a given line.*

PRO-

PROPOSITION LXIV. *Fig.* 24.

Let there be a circle given by poſition, and AB, AC *two right lines given by poſition, and let the angle* BAC *be equal to two angles of an equilateral triangle, and let the circle be contained within the angle* BAC; *a point* D *may be found, ſuch, that if through the point* D *there be drawn any line meeting the circle in* E, F, *and from the point* E *there be drawn* EG, EH *perpendicular to* AB, AC, *and likewiſe there be drawn* FK, FL *perpendicular to* AB, AC, *the ſum of the cubes of* EG, EH, *will be to the ſum of the cubes of* FK, FL, *as the ſquare of* DE *to the ſquare of* DF.

There are many properties of the circle and conic ſections ſimilar to theſe, that will naturally occur to ſuch as conſider ſome of the foregoing propoſitions, and that may be of conſiderable uſe in ſolving ſeveral problems, that at firſt view would ſeem to require a line of a high order, when the ſolution may be eaſily had by a conic ſection.

For example, *Fig.* 20. Let there be a circle given by poſition, and two points A, B given; and let it be required to draw from the given points A, B right lines to a point G in the circumference of the circle, ſuch, that the rectangle AGB may be equal to a given ſpace.

The ſolution of this problem, at firſt view, would ſeem to require the Caſſinian curve, a line of the fourth order; but it may be ſolved by the interſection of an ellipſe and circle.

Becauſe [61.] two right lines DE, DF are given by poſition, ſuch, that if from the given points A, B there be drawn right lines to any point G in the circle, and from the point G there be drawn GH, GK perpendicular to DE, DF, the ſum of the ſquares of GH, GK, will be to the rectangle AGB as the rectangle AGB to a given ſpace; and becauſe the rectangle AGB is equal to a given ſpace, the ſum of the ſquares of GH, GK, will be equal to a given ſpace: Therefore the point G will be in a given ellipſe. And therefore the point G may be found by the interſection of a given ellipſe and circle.

If the given points A, B be in the diameter, or if they be equally diſtant from the centre, or if the rectangle contained by their diſtances

Plate

Fig. 19.

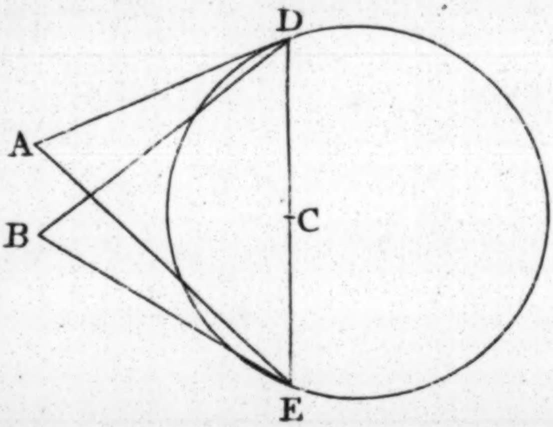


Fig. 2

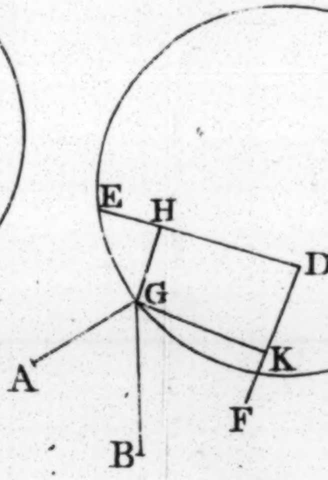


Fig. 22

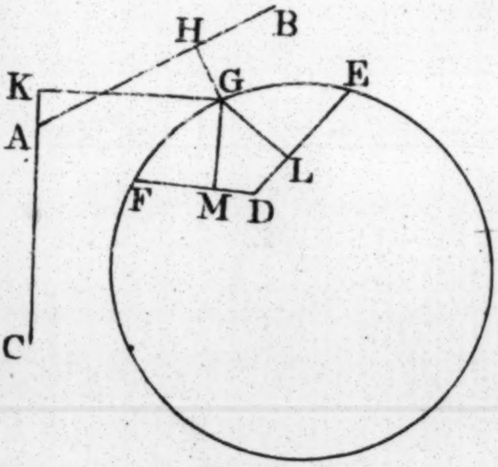


Fig. 2

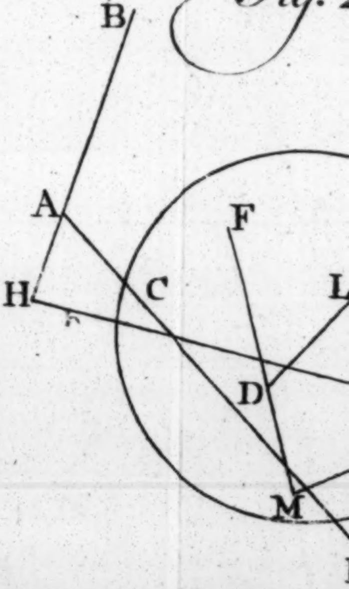


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Fig. 20

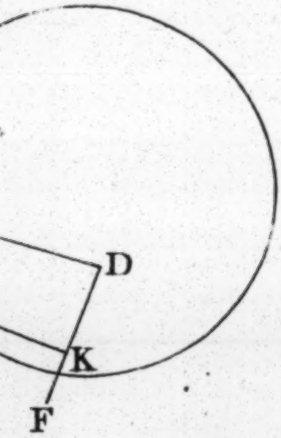


Fig. 21

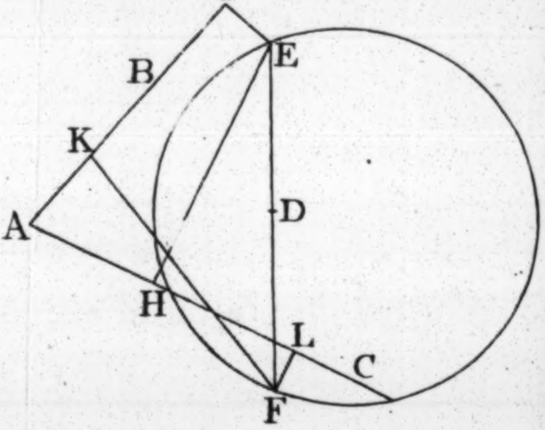


Fig. 23

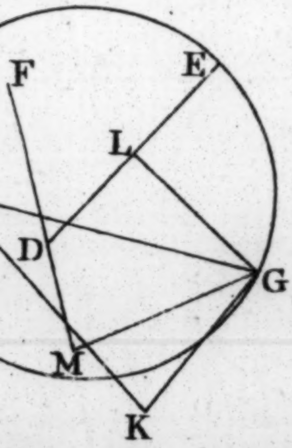
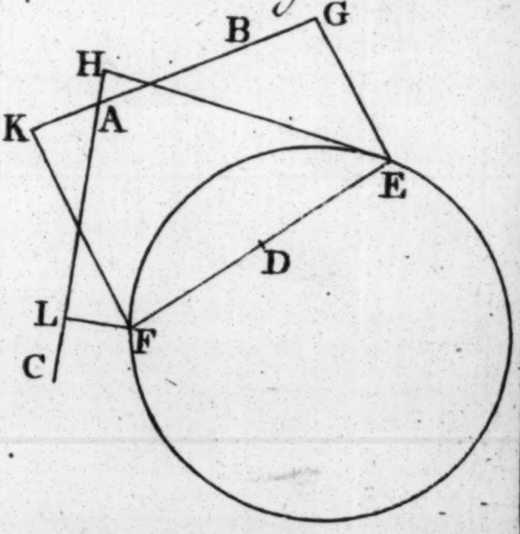


Fig. 24



from the centre be equal to the ſquare of the ſemidiameter of the circle, the point G may be found by the interſection of a right line and circle.

This problem has both a maximum and minimum, if none of the given points be in the circle; but if one of the given points be in the circle, it has only a maximum.

This problem may be ſolved otherwiſe by the ſixth propoſition.

From this it is evident, that a circle can interſect the Caſſinian curve in no more than four points.

FINIS.